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Devitoboundary: An Open-Source Tool for Topography Implementation in Finite-Difference Wavesolvers with Devito

1. What is Devito?

- **Devito is a Domain Specific Language (DSL) and compiler for implementing stencil computations** [1][2].
- Embedded in Python for integration with popular scientific packages such as NumPy, SciPy, and Dask
- High-level symbolic problem specification based on SymPy
- Generates optimized, production grade, parallel-ized C code using SIMD, OpenMP and MPI
- Multiple architectures supported including Xeon and Xeon Phi, ARM, and GPUs
- Fully open source (MIT license)

```
from devito import *

grid = grid(shape=(nx, ny), extent=(Lx, Ly))

u = TimeFunction(name='u', grid=grid, space_order=2)
u.data[0, :] = initial_data[:]

eq = Eq(u.dt, a*u.laplace)
stencil = solve(eq, u.forward)

op = Operator(Eq(u.forward, stencil))
op(t=timesteps, dt=dt)
```

Listing 1: A simple 2D diffusion operator in Devito. The symbolic specification lends itself to ease of use, code readability, and accelerated workflow. The FD operator itself is encapsulated in the Operator object.

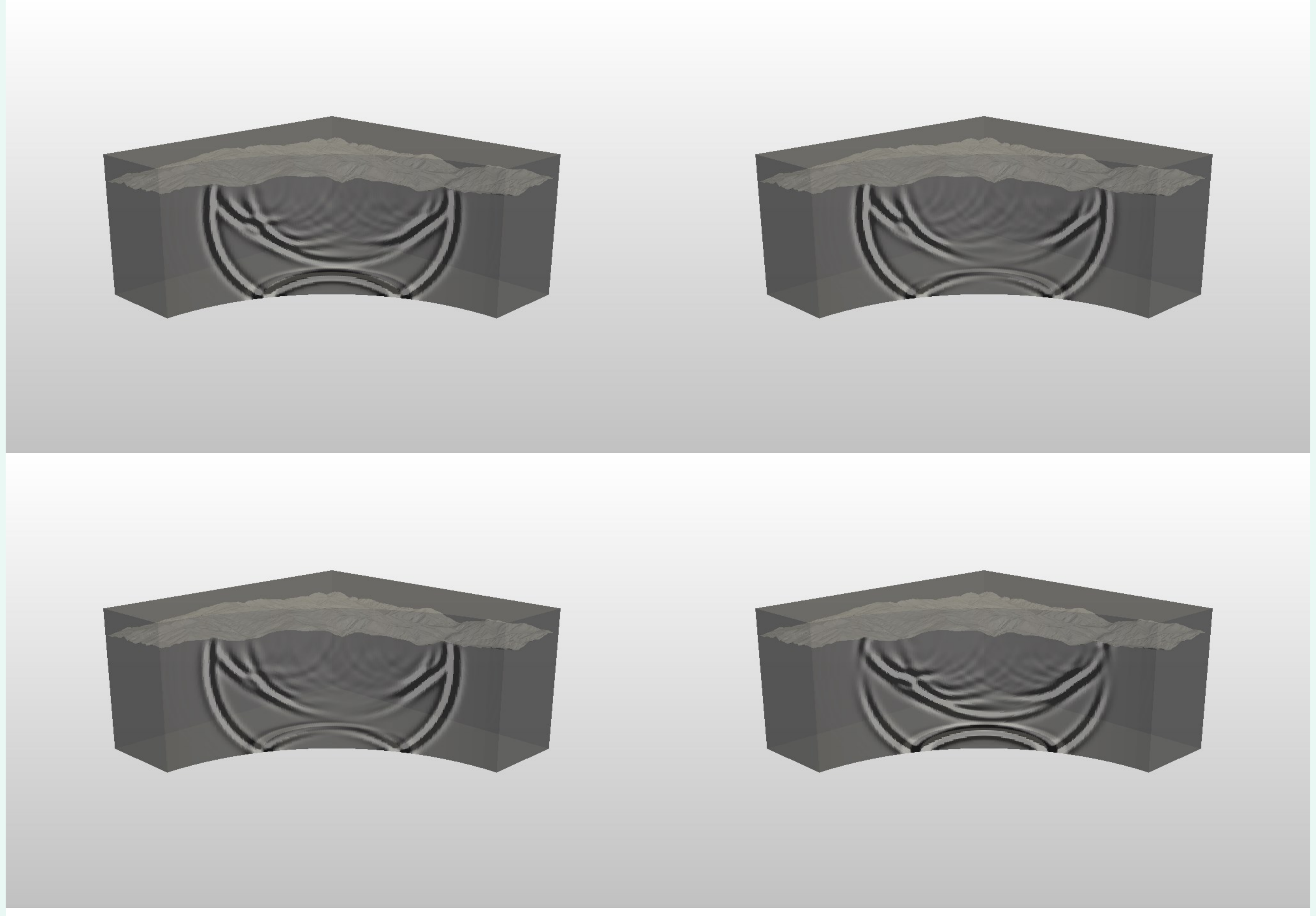

Figure 2: A cutaway render at t=4s of the wavefields for the topographic scattering model. Counter-clockwise from top left, the renders show pressure, followed by particle velocities in the x, y, and z directions respectively. The model run demonstrated expected scattering behaviour with only minor distortion of the outgoing wavefront (due to diffraction around the surface), but a complex series of reflections and reverberations following.

2. Topography in Seismic Modelling

- **Seismic imaging/modelling scenarios such as FWI and migration, may require complex topography to be accounted for within the numerical scheme**
- In imaging applications, failure to account for topography can produce unrealistic propagation patterns, degrading image quality[3] requiring careful representation of the free surface[4]
- Topography represents a sharp, irregular discontinuity which is difficult to include in wave propagators based on structured grids, such as finite-difference (FD) solvers[4][5][6]
- FD is commonly used in seismic applications as it is conceptually simple, relatively computationally cheap, and has a suite of known optimizations[1]
- Naïve 'vacuum-layer' approaches, whilst straightforward have poor stability characteristics and generate spurious scattering artifacts[5]
- We wish to accurately represent complex topography whilst retaining the advantages of structured grids

3. Immersed Boundaries

- **Used to impose boundary conditions on smooth surfaces of arbitrary shape within FD schemes, without geometric transformations**
- Pioneered in fluid-flow simulations[7]
- Difference approximations based on polynomials fitted using free-surface boundary conditions in addition to interior points
- As the additional boundary points where the function is fitted do not need to be coincident with a grid node, boundary conditions can be imposed off-grid whilst retaining a single cartesian grid

4. Devitoboundary

- **Devitoboundary is an open-source add-on for Devito which aims to provide a high-level interface for including immersed boundaries in seismic applications**
- Immersed boundary is encapsulated in a handful of high-level objects
- Integrates with Devito's custom coefficients functionality

5. Method

- **Immersed boundary implemented using variable stencil coefficients in the vicinity of the boundary, removing the need for a ghost grid**
- Boundary surface represented as a signed distance function (SDF) generated from 3D mesh discretized onto FD grid
- Lagrange polynomials of equal order to the FD scheme fitted to at interior stencil points and boundary points identified by the SDF
- Independent 1D extrapolation per coordinate direction automatically generated using symbolic computation
- Extrapolated exterior values are substituted into the FD stencils

6. Mountainous Model

- **Forward model based on 1st-order formulation of the acoustic wave equation to demonstrate modelling of topographic effects**
- Zero even pressure derivatives at the free-surface
- Discretization is 4th-order accurate in space and 2nd-order accurate in time
- 10.8km x 10.8km x 5.4km FD grid with Δx=50m
- Central Ricker source positioned 500m below sea level, injected into pressure field
- Homogenous material properties, meaning that all scattering observed is a product of the boundary treatment

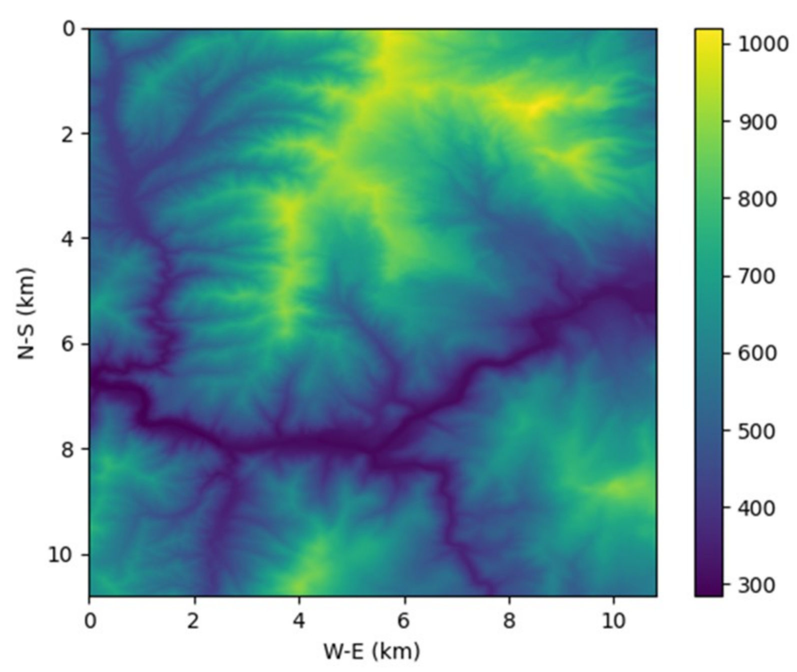

Figure 1: Surface geometry used in the scattering model, taken from 1-arcsecond SRTM DEM of a mountainous area of Oregon. Scalebar shows elevation in meters.

7. Convergence

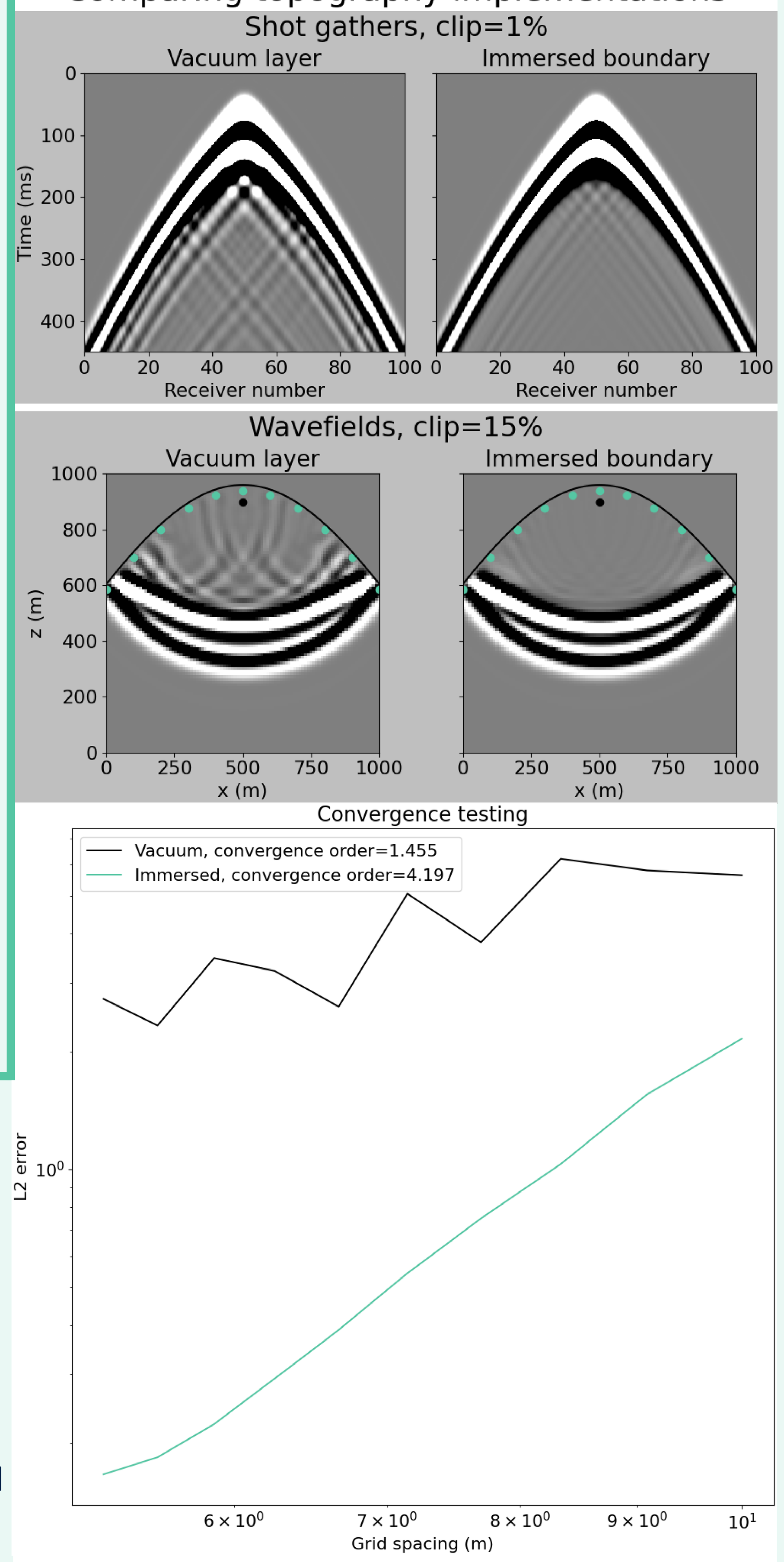

Figure 3: A comparison between vacuum layer and immersed boundary implementations. Black and teal dots are sources and decimated receivers respectively. The vacuum layer exhibits strong scattering artifacts following the reflected wave, which are suppressed in the immersed boundary model, resulting in much cleaner reflected geometry. Convergence testing demonstrates the superior convergence for the immersed boundary approach.

- **Immersed boundary accuracy compared to a naïve vacuum-layer approach for modelling reflections from a simple hill**
- 2nd-order acoustic wave propagator with homogenous material properties
- Discretization is 4th-order accurate in space and 2nd-order accurate in time
- Figure 3 compares wavefields and gathers
- 8th-order immersed-boundary run on twofold -refined grids used as benchmark
- The immersed-boundary method achieved specified 4th-order convergence
- Vacuum methods of equivalent order exhibited an order of magnitude more error, and inconsistent convergence with a sub-2nd-order trend

8. Ongoing Work

- **We are currently working on a novel immersed boundary approach enabling handling of vector and tensor boundary conditions**
- This will improve accuracy and stability for the 1st-order acoustic wave equation and enable immersed boundaries to be implemented for the elastic wave equation
- Figure 4 shows results from a 2nd-order-accurate, isotropic elastic prototype, containing a horizontal immersed boundary located off-grid at 0.75Δx above the node
- Method is straightforwardly generalisable to higher orders and dimensions
- Method is nominally extendable to anisotropic forms of the elastic wave equation

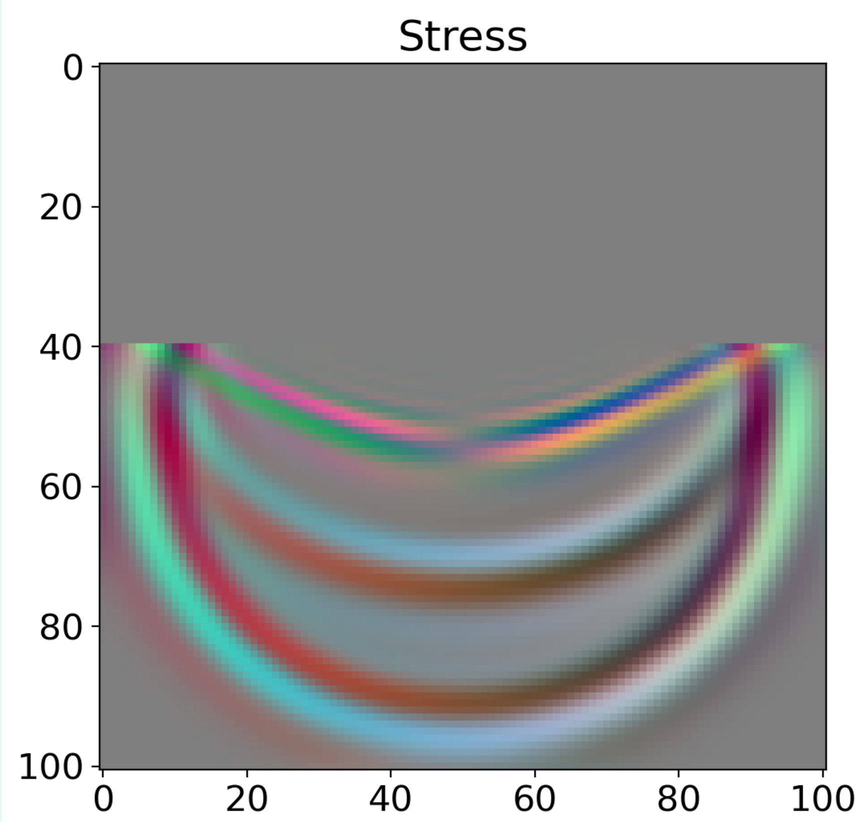

Figure 4: An RGB plot of the stress fields in a 2nd-order-accurate isotropic elastic model containing an immersed boundary. The red, green, and blue channels correspond to τ_{xy}, τ_{xx}, and τ_{yy} respectively.

Contact and Resources

Email - Website - Github -

References

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