Imperial College London

TEMPORAL BLOCKING FOR WAVE-PROPAGATION KERNELS WITH SPARSE OFF-THE-GRID SOURCES

George Bisbas[1], **Fabio Luporini**[2], **Mathias Louboutin**[3], **Rhodri Nelson**[1], **Gerard Gorman**[1], **Paul Kelly**[1]

[1]**Imperial College London**
[2]Devito Codes
[3]Georgia Tech

Oil and Gas

High-Performance Computing Conference 2021

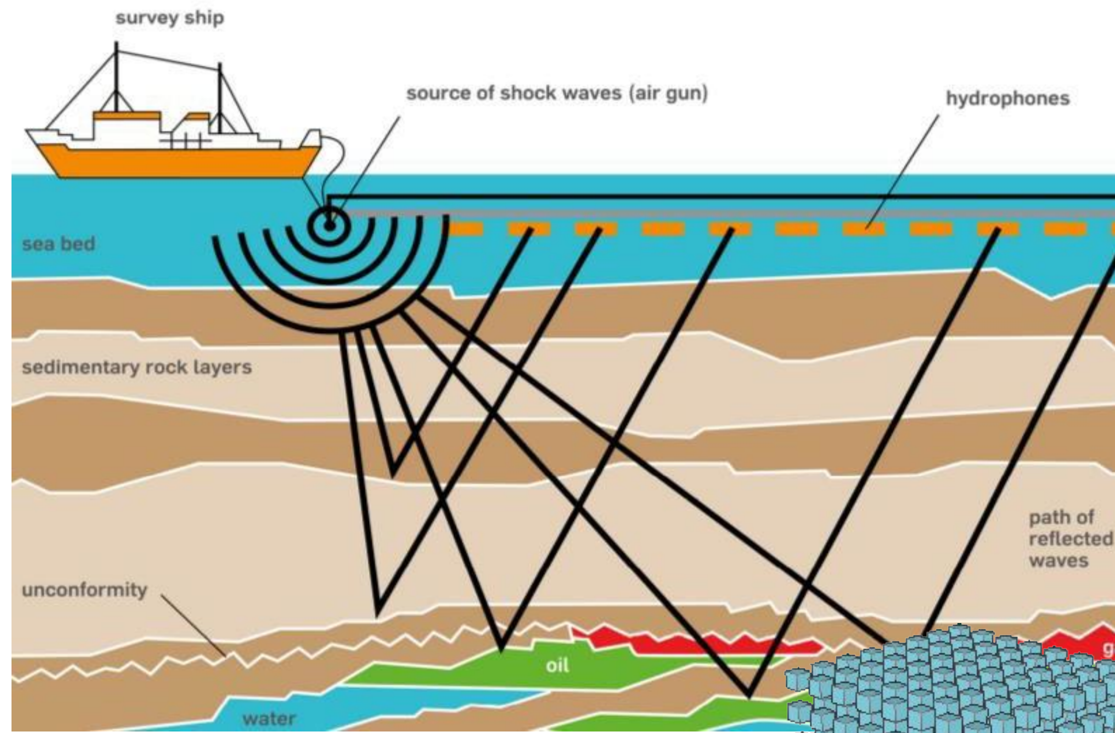
Rice University, Ken Kennedy Institute, Houston

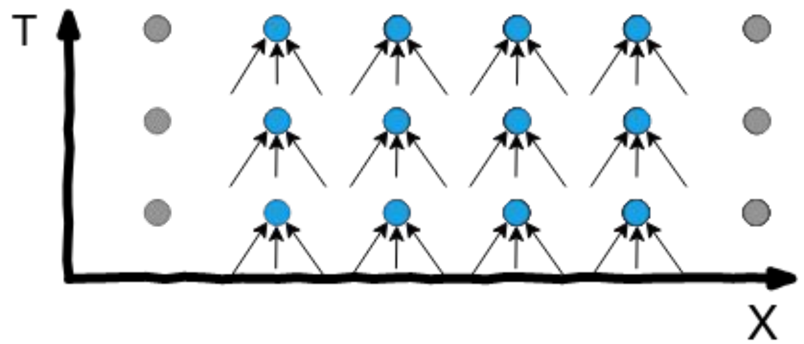
Motivation

- Accelerating wave-propagation kernels of practical interest through cache optimizations, more specifically through temporal blocking
- Enabling **Temporal Blocking** on practical wave-propagation simulations is complicated
- They consist of **sparse "off-the-grid"** operators (**Not** a typical stencil benchmark!)
- Applicability issues due to seismic imaging kernels consisting of sparse off-the-grid operators
- We present an approach to overcome limitations and enable TB
- Experimental results show improved performance

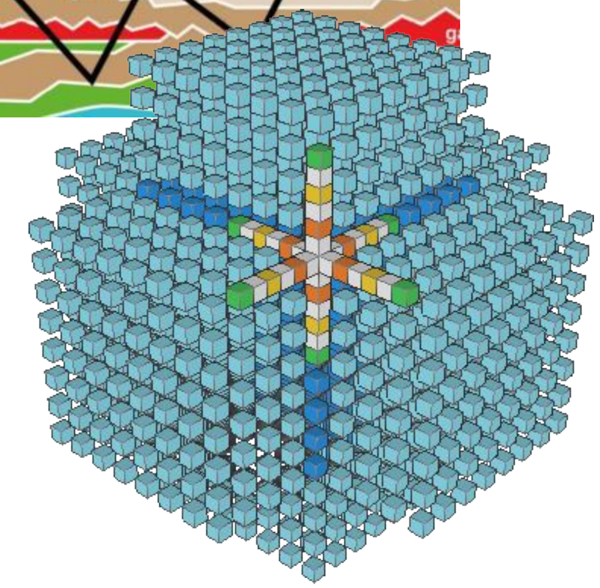
Modelling practical applications

- Stencils everywhere, but not only. What else?
- Vast literature on optimizing stencils… (Parallelism, cache optimizations, accelerators)
- Sources injecting and receivers interpolating at sparse off-the-grid coordinates. **Non-conventional update patterns.**
- Usually their coordinates are not aligned with the computational grid. How do we iterate over them?

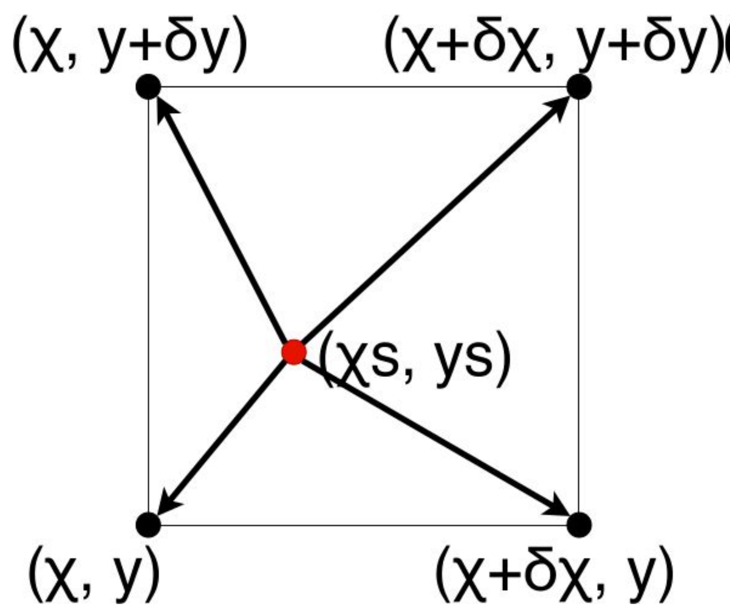


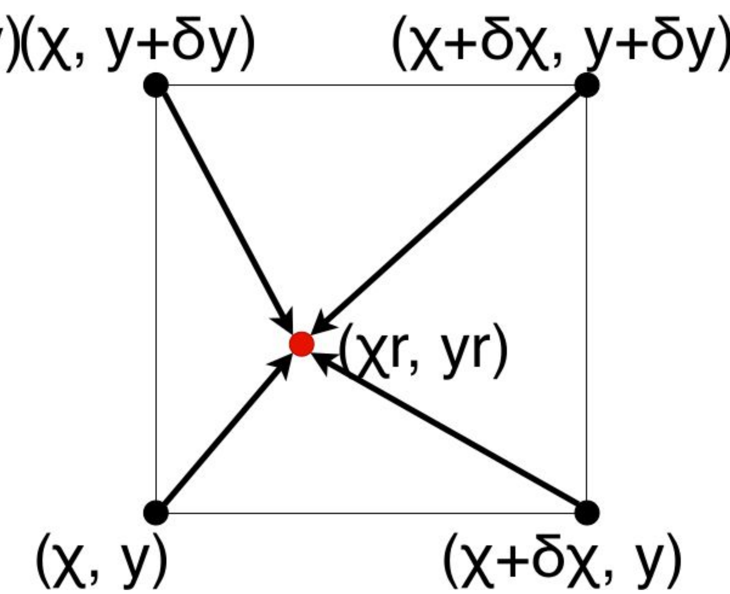


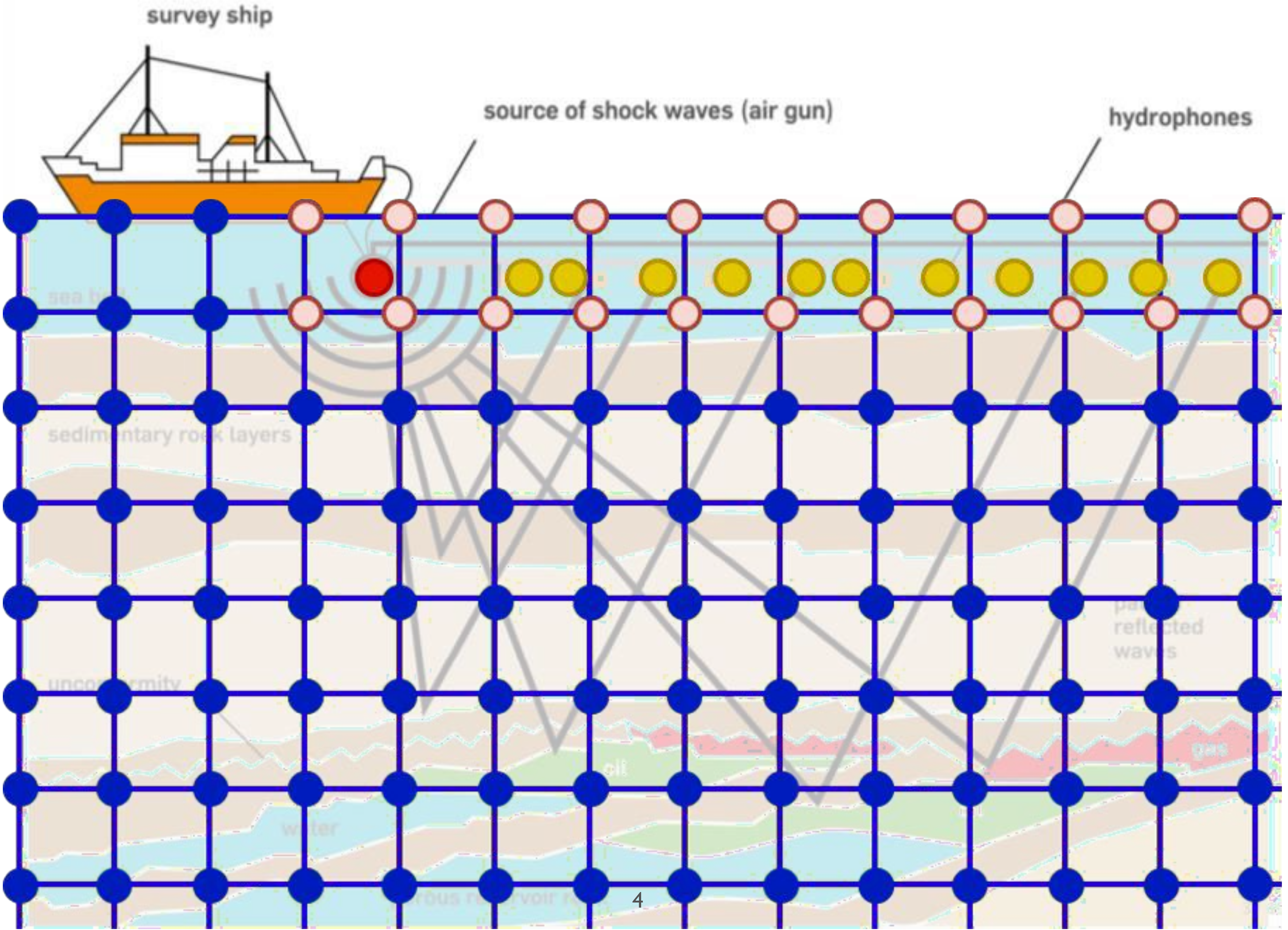
A 1d 3pt stencil update



A 3d-19pt stencil update






survey ship
source of shock waves (air gun)
hydrophones

A typical time-stepping loop with source injection

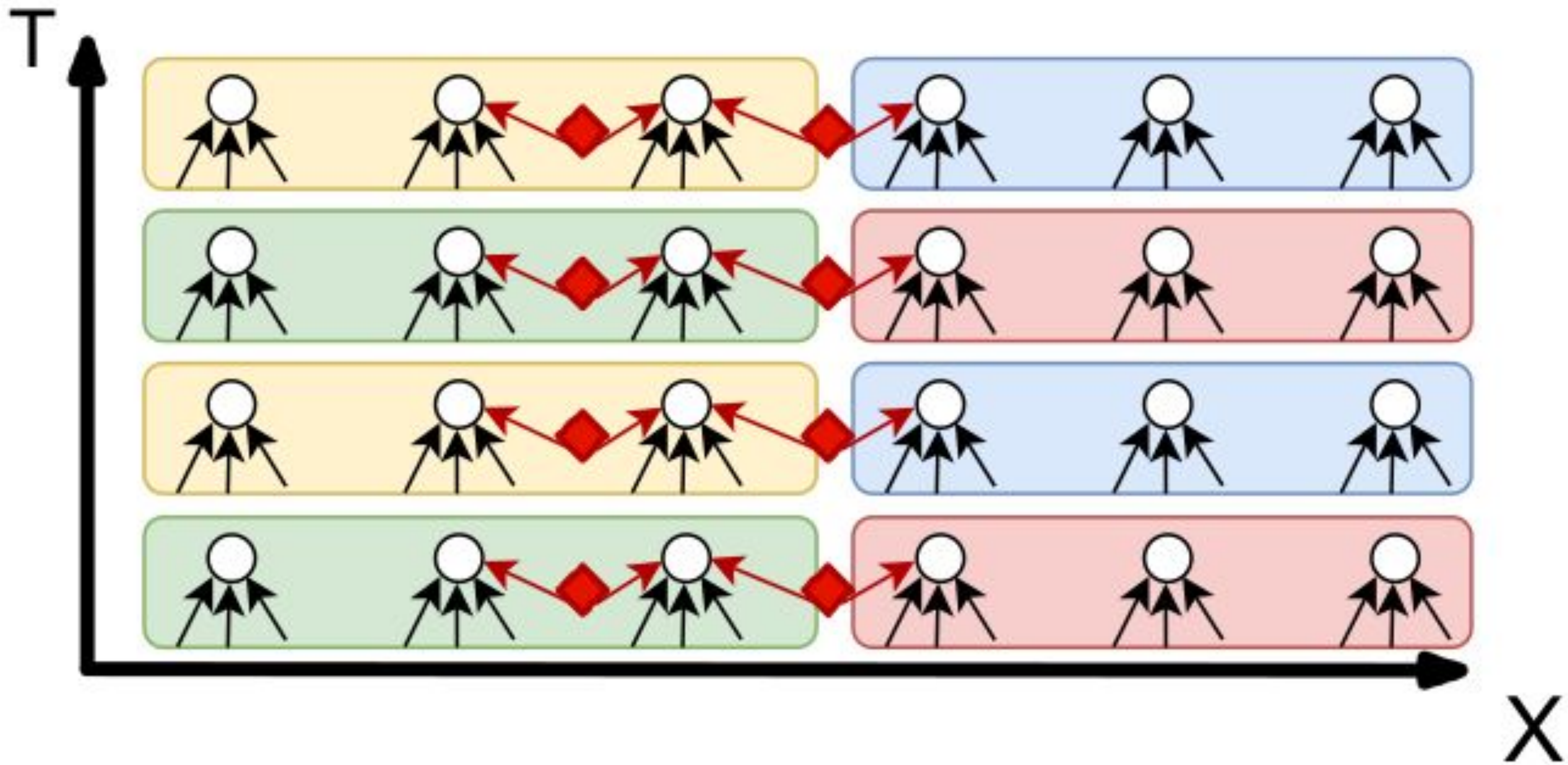
- Iterate over sources, each has 3-d coordinates
- Indirect accesses to scatter injection to neighbouring points
- Aligned in time, not in space

Listing 1: A typical time-stepping loop nest structure for a stencil update with source injection. This stencil has one temporal and three spatial dimensions.

for t = 1 ***to*** nt **do**
  for x = 1 ***to*** nx **do**
    for y = 1 ***to*** ny **do**
      for z = 1 ***to*** nz **do**
        $A(t, x, y, z) \equiv$ u[t, x, y, z] = u[t-1, x, y, z] + $\sum_{r=1}^{r=so/2} w_r \Big($ u[t-1, x - r, y, z] + u[t-1, x + r, y, z] + u[t-1, x, y - r, z] + u[t-1, x, y + r, z] + u[t-1, x, y, z - r] + u[t-1, x, y, z + r] $\Big)$;
  foreach s ***in*** sources **do** // For every source
    for i = 1 ***to*** np **do** // Get the points affected
      xs, ys, zs = map(s, i) // through indirection
      u[t, xs, ys, zs] + = $f(src(t, s))$ // add their impact on the field

Applying space-blocking

- Spatial blocking:
  - Decompose grids into block tiles/ Partitioning iteration space to smaller chunks/blocks
  - Improve data locality => Increase performance (Rich literature)
  - Sparse off-the-grid operators are iterated as without blocking

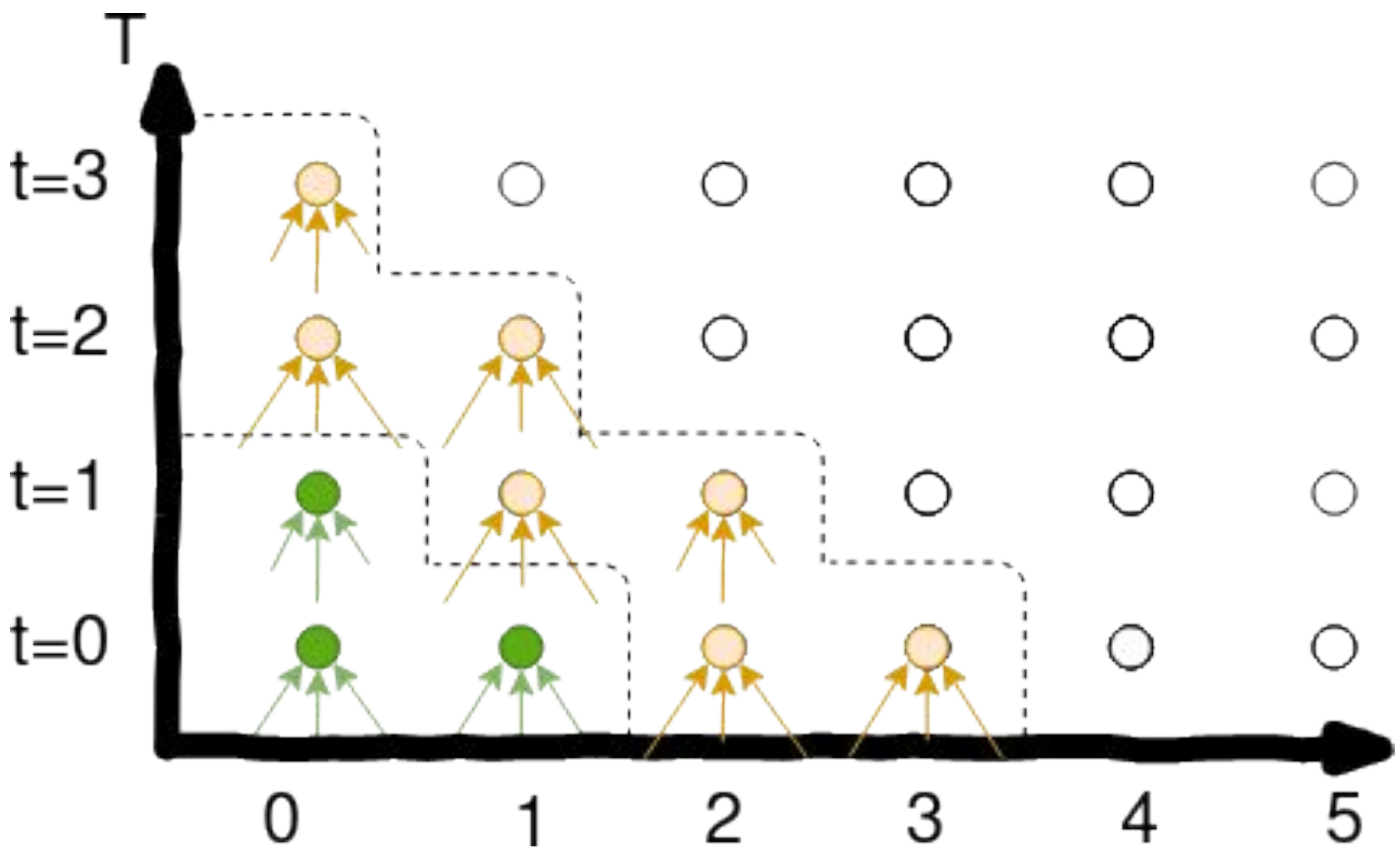


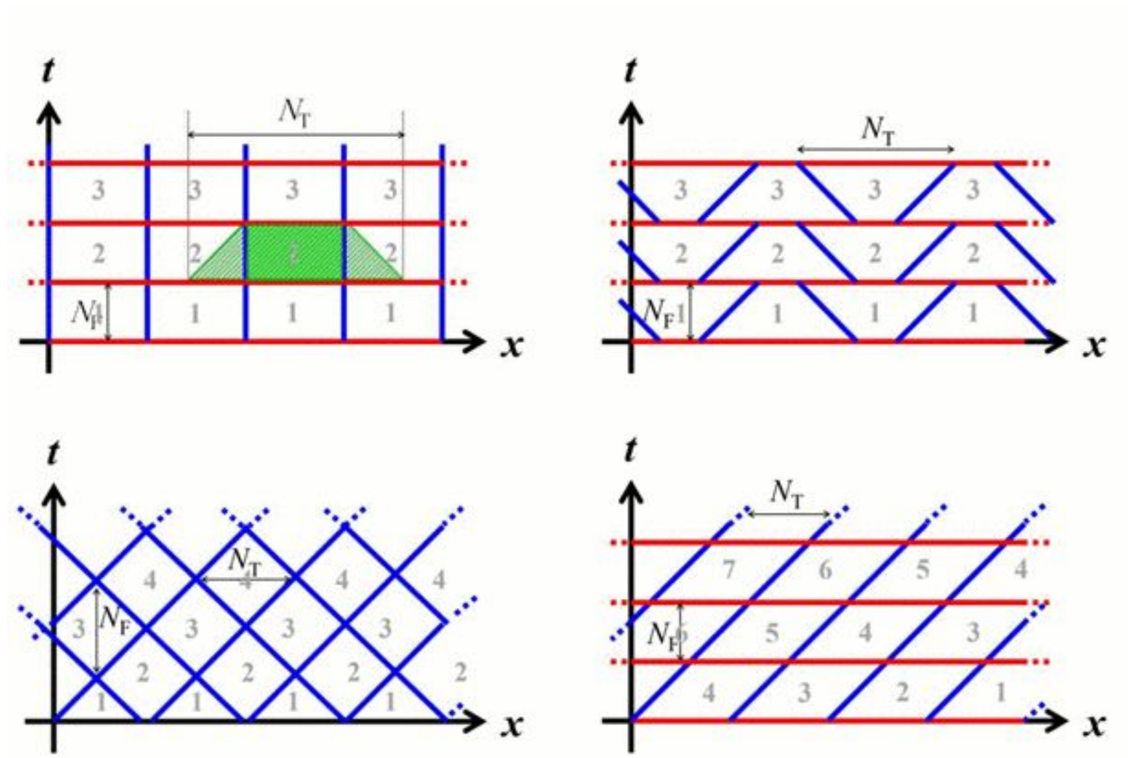
Applying temporal-blocking

- Temporal blocking:
  - Space blocking but extend reuse to time-dimension.
  - Update grid points in future where/when (space+time) possible
  - Rich literature, several variants of temporal blocking, shapes, schemes

    -Wave-front / Skewing (Our POC approach)

    -Diamonds, Trapezoids, Overlapped, Hybrid models

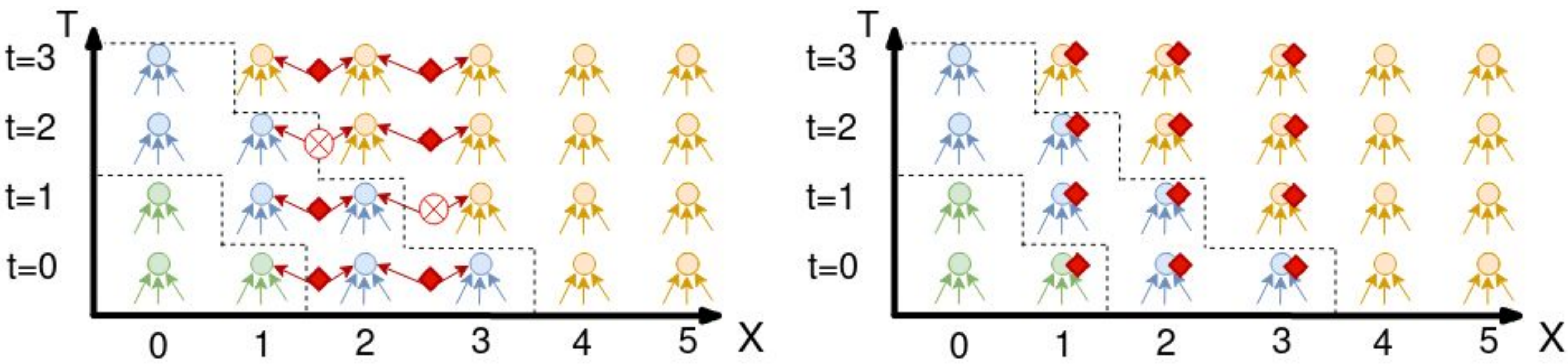




Tanaka et.al. (2018)

Off-the-grid operators: the issue

- Data dependences violations happen while a temporal update
- Source injection in a different iteration space
- When a sparse operator exists in the boundary between space-time blocks, order of updates is not preserved
- Solution: Need to align off-the-grid operators



Methodology

- A negligible-cost scheme to precompute the source injection contribution.

- Align source injection data dependences to the grid

- Applicable to other fields as well

Iterate over sources and store indices of affected points

- Inject to a **zero-initialized grid** for one (or a few more)
- **Hypothesis:** non-zero values at the first time-steps
- **Independent** of the injection and interpolation type (e.g. non-linear injection)

Listing 2: Source injection over an empty grid. No PDE stencil update is happening.

```
for t = 1 to 2 do
    foreach s in sources do
        for i = 1 to np do
            xs, ys, zs = map(s, i);
            u[t, xs, ys, zs] + = f(src(t, s))
```

- Then, we store the non-zero grid point coordinates

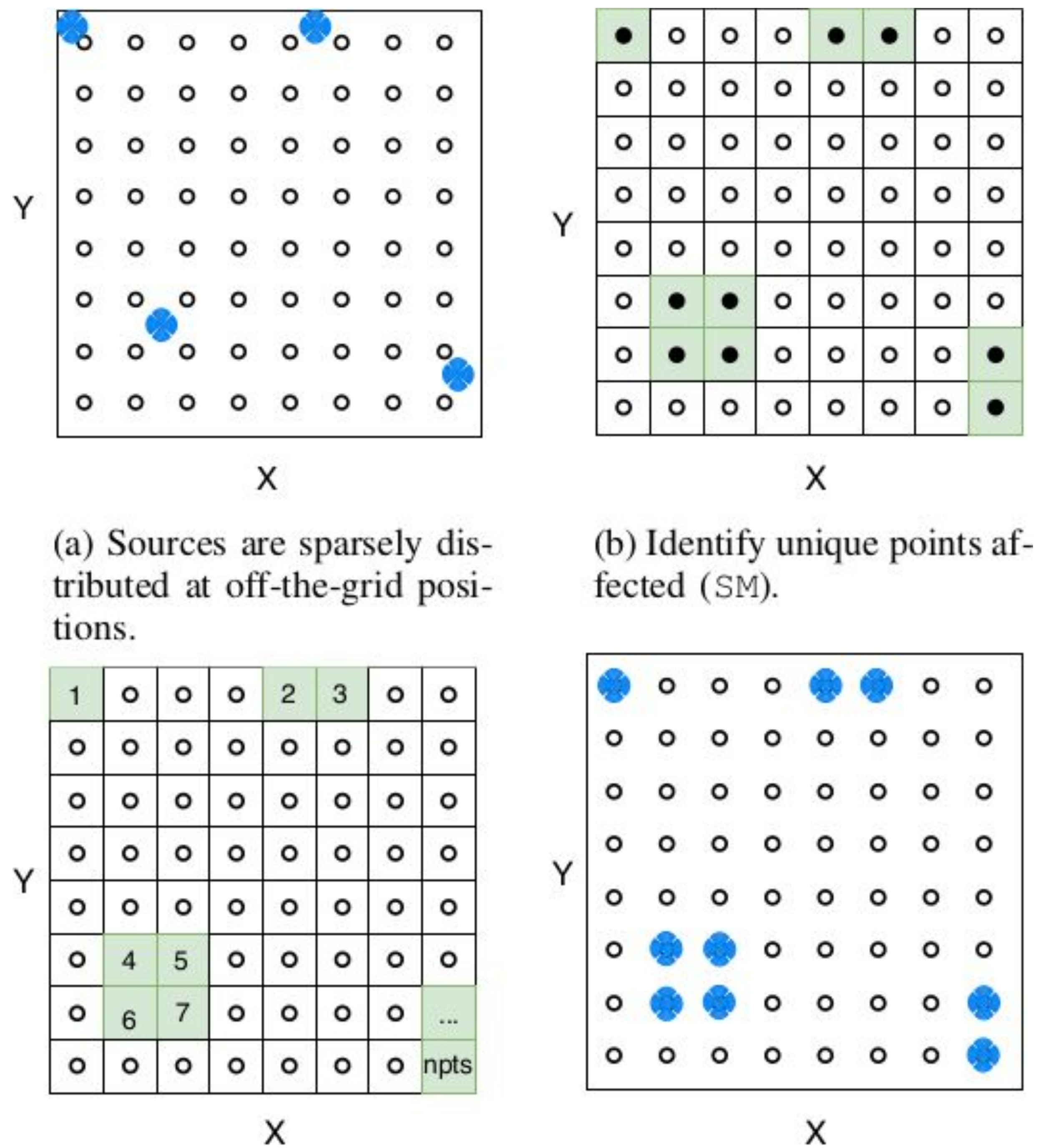
Generate sparse binary mask, unique IDs and decompose wavefields

Perform source injection to decompose the off-the-grid wavefields to on-the-grid per point wavefields.

	Off-the-grid	Aligned
len(sources)	n_src	n_aff_pts
len(sources.coords)	(n_src, 3)	(n_aff_pts, 3)
len(sources.data)	(n_src, nt)	(n_aff_pts, nt)

Listing 3: Decomposing the source injection wavefields.

```
for t = 1 to nt do
    foreach s in sources do
        for i = 1 to np do
            xs, ys, zs = map(s, i);
            src_dcmp[t, SID[xs, ys, zs]] + = f(src(t, s));
```



(a) Sources are sparsely distributed at off-the-grid positions.

(b) Identify unique points affected (SM).

(c) Assign a unique ID to every affected point (SID).

(d) Sources are aligned with grid positions.

Fuse iteration spaces

- Indirection mapping has changed. We still use indirections but now they are on the point.
- By using the aligned structure, we fuse the source injection loop inside the kernel update iteration space.
- The source mask SM is used to add (if 1) or not (if 0) the impact and SID is used to indirect to the impact values using the traversed grid coordinates.

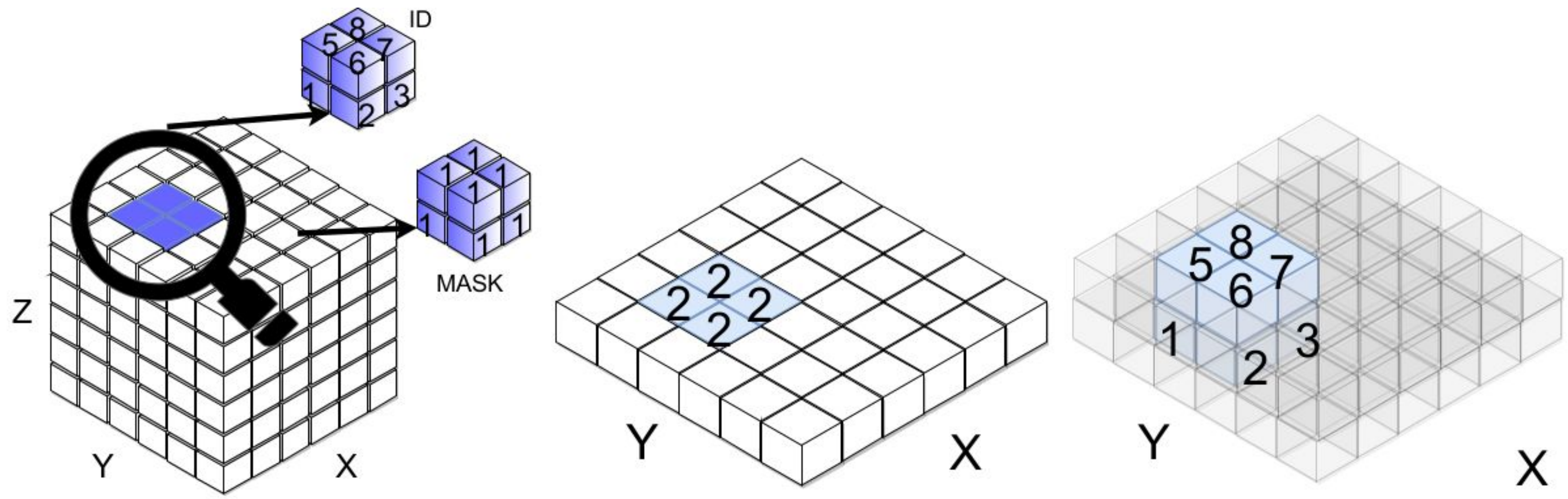
Listing 4: Stencil kernel update with fused source injection.

```
for t = 1 to nt do
    for x = 1 to nx do
        for y = 1 to ny do
            for z = 1 to nz do
                A(t, x, y, z, s);
            for z2 = 1 to nz do
                u[t, x, y, z2] + = SM[x, y, z2] * src_dcmp[t, SID[x, y, z2]];
```

SIMD? (AVX512)

Reducing the iteration space size

- Perform only necessary operations
- Aggregate NZ along the z- axis keeping count of them in a structure named *nnz_mask*
- Reduce the size of SID by cutting off zero z-slices



(a) `SID` and `SM` are very sparse in the general case.

(b) `nnz_mask` Aggregating non-zero values along z-axis.

(c) `Sp_SID`, a reduced size `SID`.

Listing 5: Stencil kernel update with reduced size iteration space for source injection.

for t = 1 ***to*** nt **do**
  for x = 1 ***to*** nx **do**
    for y = 1 ***to*** ny **do**
      for z = 1 ***to*** nz **do**
        $A(t, x, y, z, s)$;
      for $z2$ = 1 ***to*** nnz_mask[x][y] **do**
        $I(t, x, y, z, s) \equiv$\{ zind = Sp_SID[x, y, z2];
        u[t, x, y, z2] += src_dcmp[t, SID[x, y, zind]]; \}

Listing 1: A typical time-stepping loop nest structure for a stencil update with source injection. This stencil has one temporal and three spatial dimensions.

for t = 1 ***to*** nt **do**
  for x = 1 ***to*** nx **do**
    for y = 1 ***to*** ny **do**
      for z = 1 ***to*** nz **do**
        $A(t, x, y, z) \equiv$ u[t, x, y, z] = u[t-1, x, y, z] + $\sum_{r=1}^{r=so/2} w_r \Big($ u[t-1, x - r, y, z] + u[t-1, x + r, y, z] + u[t-1, x, y - r, z] + u[t-1, x, y + r, z] + u[t-1, x, y, z - r] + u[t-1, x, y, z + r] $\Big)$;
  foreach s ***in*** sources **do** // For every source
    for i = 1 ***to*** np **do** // Get the points affected
      xs, ys, zs = map(s, i) // through indirection
      u[t, xs, ys, zs] + = $f(src(t, s))$ // add their impact on the field

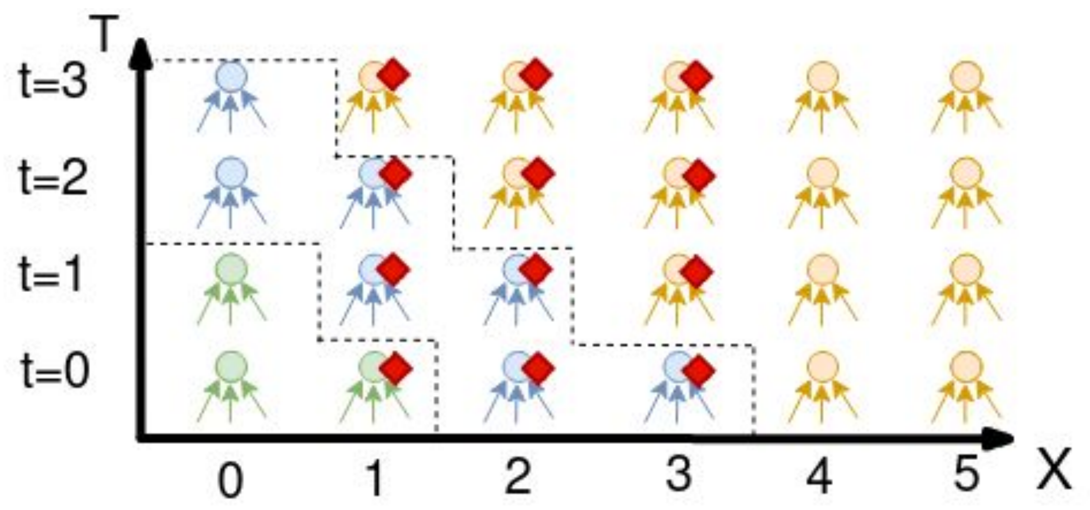
Non-aligned

- Aligned to grid
- Same OPS
- Parallelism
- SIMD
- Apply TB

Listing 5: Stencil kernel update with fused - reduced size iteration space - source injection.

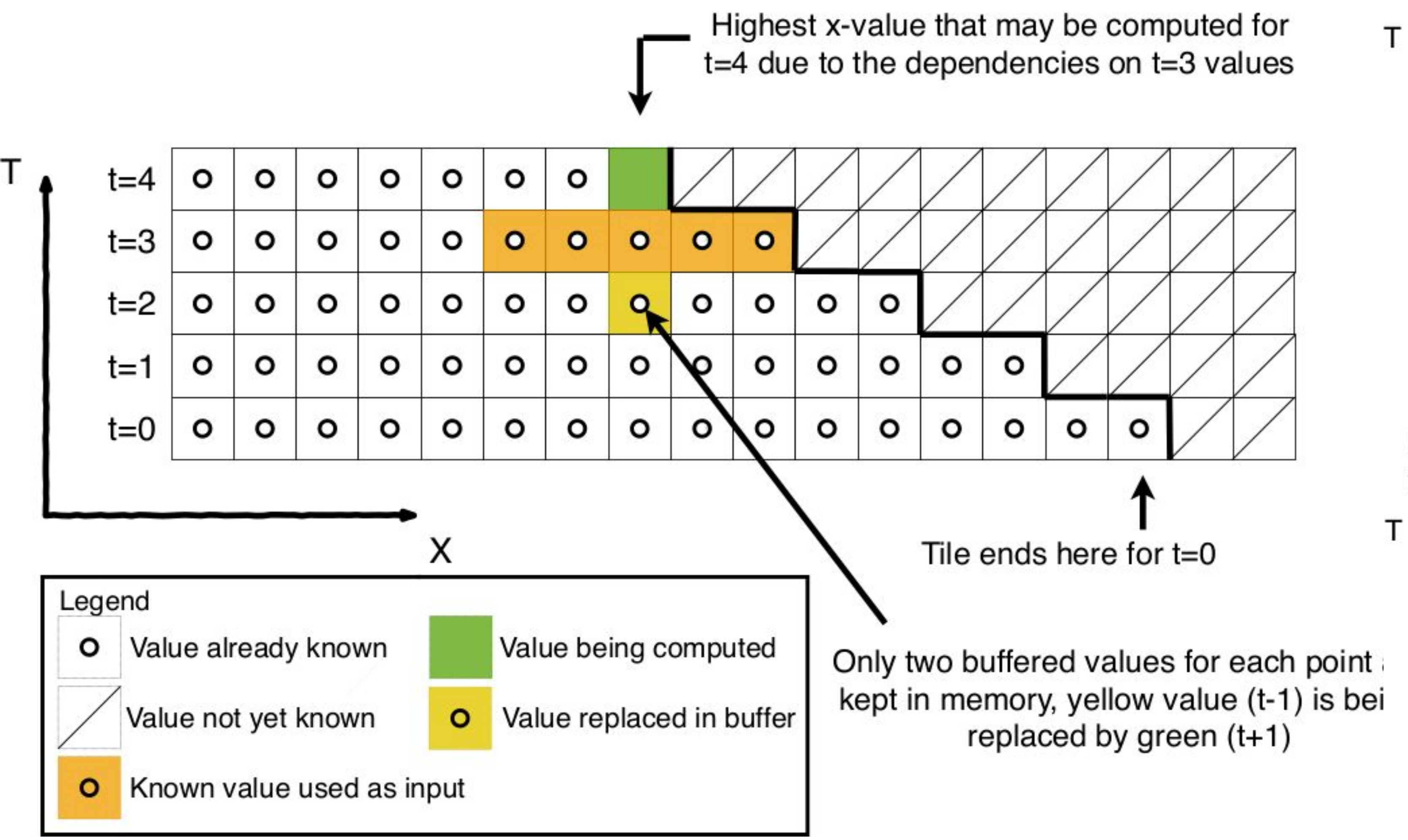
for t = 1 ***to*** nt **do**
  for x = 1 ***to*** nx **do**
    for y = 1 ***to*** ny **do**
      for z = 1 ***to*** nz **do**
        $A(t, x, y, z, s)$;
      for $z2$ = 1 ***to*** nnz_mask[x][y] **do**
        $I(t, x, y, z, s) \equiv$ { zind = Sp_SM[x, y, z2];
        u[t, x, y, z2] +=
        SM[x, y, zind] * src_dcmp[t, SID[x, y, zind]]; }

Aligned



Applying wave-front temporal blocking

- Aligning, automated in Devito DSL; TB with manual loop transformation
- The new source coordinates are aligned to the grid now
- Skewing factor depends on data dependency distances



YASK, Yount et. al (2016)

(a) The figure shows multiple wave-fronts tiles evaluate sequentially, partially adapted from [15].

(b) The figure shows multiple wave-front tiles evaluated sequentially in multigrid stencil codes.

Listing 6: The figure shows the loop structure after applying our proposed scheme.

```
for t_tile in time_tiles do
    for xtile in xtiles do
        for ytile in ytiles do
            for t in t_tile do
                OpenMP parallelism
                for xblk in xtile do
                    for yblk in ytile do
                        for x in xblk do
                            for y in yblk do
                                SIMD vectorization
                                for z = 1 to nz do
                                    A(t, x - time, y - time, z, s);
                                for z2 = 1 to nnz_mask[x][y] do
                                    I(t, x - time, y - time, z2, s);
```

Experimental evaluation: the models

- **Isotropic Acoustic**

  Generally known, single scalar PDE, laplacian like, low cost

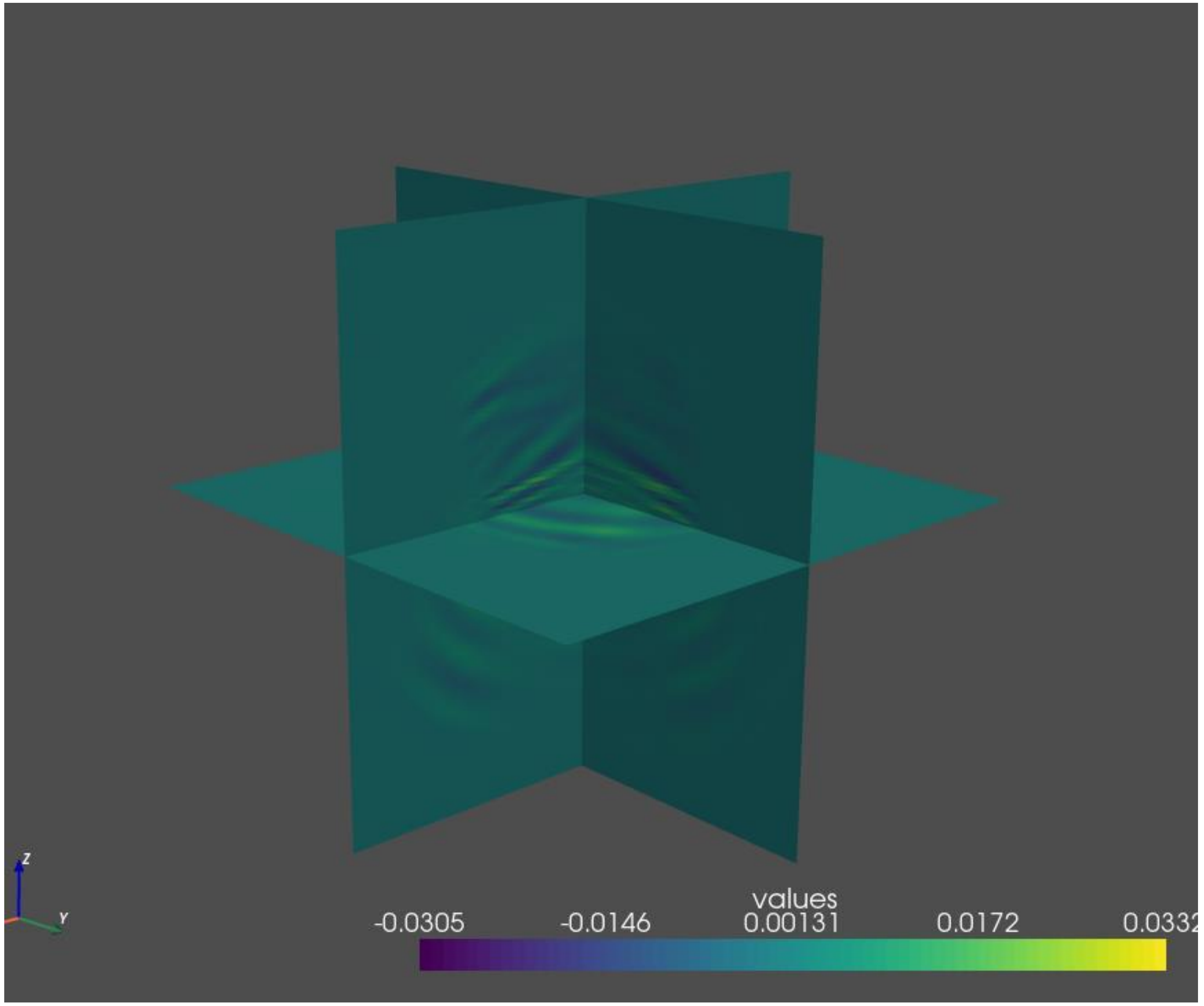
- **Isotropic Elastic**

  Coupled system of a vectorial and tensorial PDE, explosive source, increased data movement, first order in time, cross-loop data dependences

- **Anisotropic Acoustic**

  Industrial applications, rotated laplacian, coupled system of two scalar PDEs

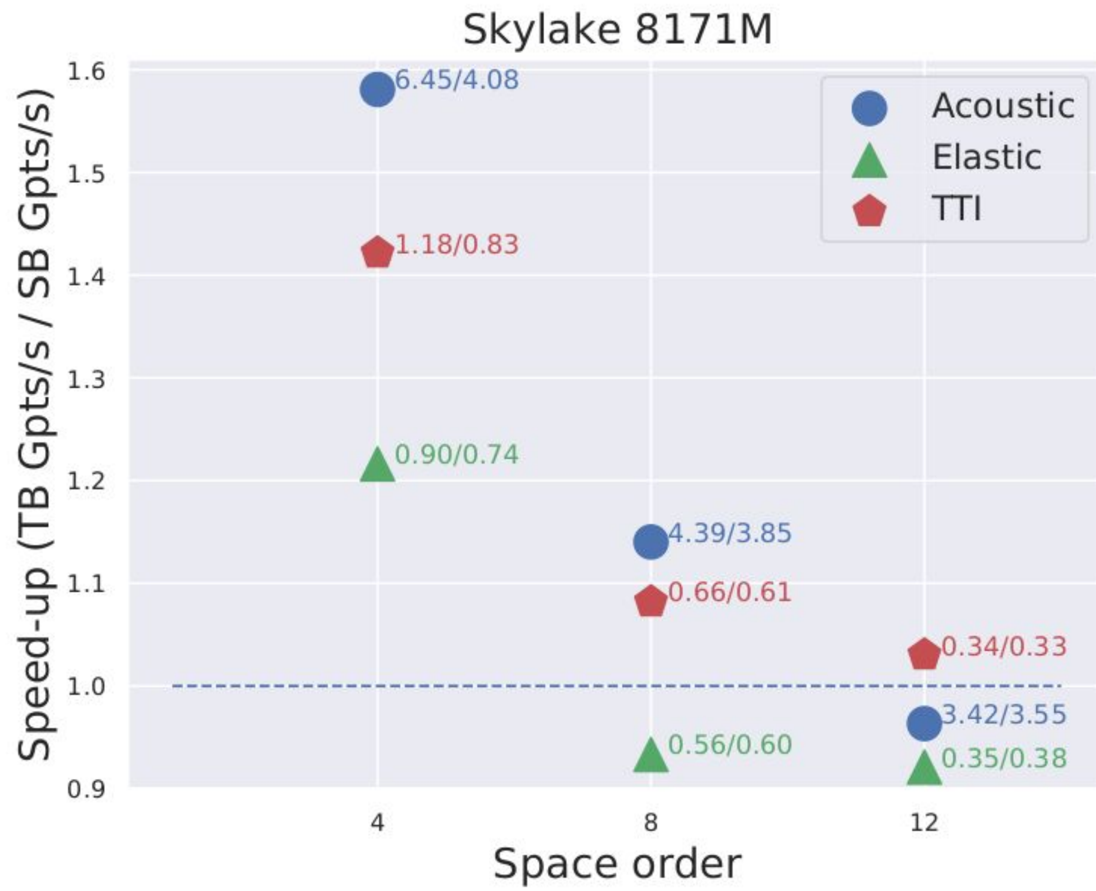
  Industrial-level, 512^3 grid points, 512ms simulation time, damping fields ABCs



Velocity field, TTI wave propagation after 512ms

Experimental evaluation: the results

(a) Throughput speed-up of kernels for Broadwell.



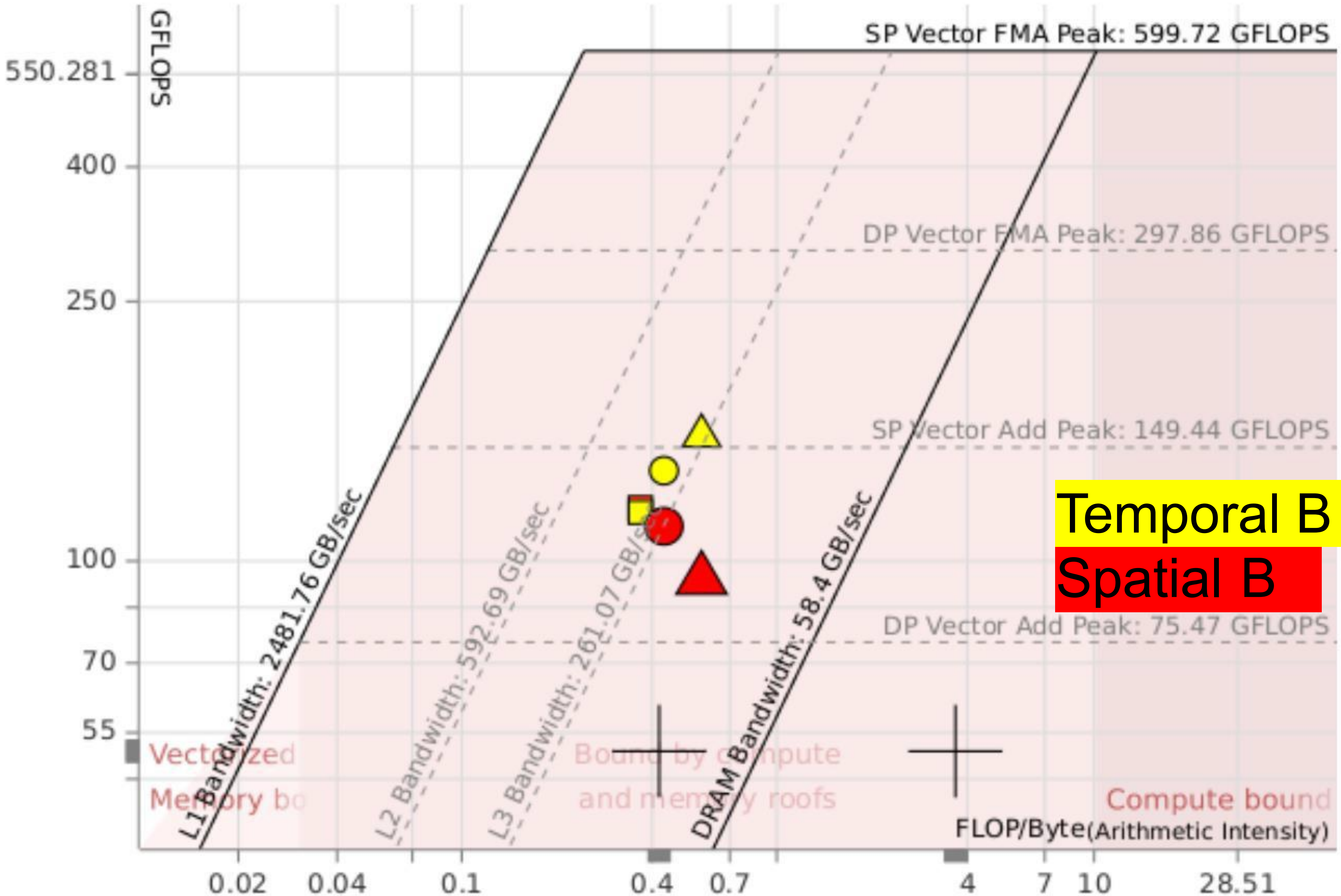
(b) Throughput speed-up of kernels for Skylake.

Azure model Architecture	**E16s v3** **Broadwell**	**E32s v3** **Skylake**
vCPUs	16	32
GiB memory	128	256
Model name	E5-2673 v4	8171M
CPUs	16	32
Thread(s) per core	2	2
Core(s) per socket	8	16
Socket(s)	1	1
NUMA node(s)	1	1
Model	79	85
CPU MHz	2300	2100
L1d cache	32K	32K
L1i cache	32K	32K
L2 cache	256K	1024K
L3 cache	51200K	36608K

TABLE I: VM specification

- Benchmark on Azure VMs
- GCC, ICC
- Thread pinning
- OpenMP, SIMD
- Aggressive auto-tuning

Cache-aware roofline model



Space order:

- △4
- ○8
- □12

Broadwell, isotropic acoustic, 512^3 grid points, 512ms

Conclusions

- We presented an approach to apply temporal blocking on stencil kernels with sparse off-the-grid operators.
- The additional cost is negligible compared to the achieved gains.
- Solution built on top of Devito-DSL
- Performance gains of up to 1.6x on low order (4) and 1.2x on medium order (8).

Work presented is inherited from: Bisbas, G., Luporini, F., Louboutin, M., Nelson, R., Gorman, G., & Kelly, P.H. (2020). Temporal blocking of finite-difference stencil operators with sparse "off-the-grid" sources. Accepted at IPDPS'21. Available online: https://arxiv.org/abs/2010.10248

Future plans ➡️

- Integration/ Automation
- GPUs
- High-order stencils

- Open-source, on top of Devito v4.2.3 - https://github.com/georgebisbas/devito

Website: http://www.devitoproject.org
GitHub: https://github.com/devitocodes/devito
Slack: https://opesci-slackin.now.sh

Imperial College London

EPSRC Engineering and Physical Sciences Research Council

HiPEDS

Acknowledgements

Thanks to collaborators and contributors:

- Navjot Kukreja (Imperial College)
- John Washbourne (Chevron)
- Edward Caunt (Imperial College)

Thank you for your attention!
Questions?

Imperial College London

EPSRC
Engineering and Physical Sciences Research Council

HiPEDS

References

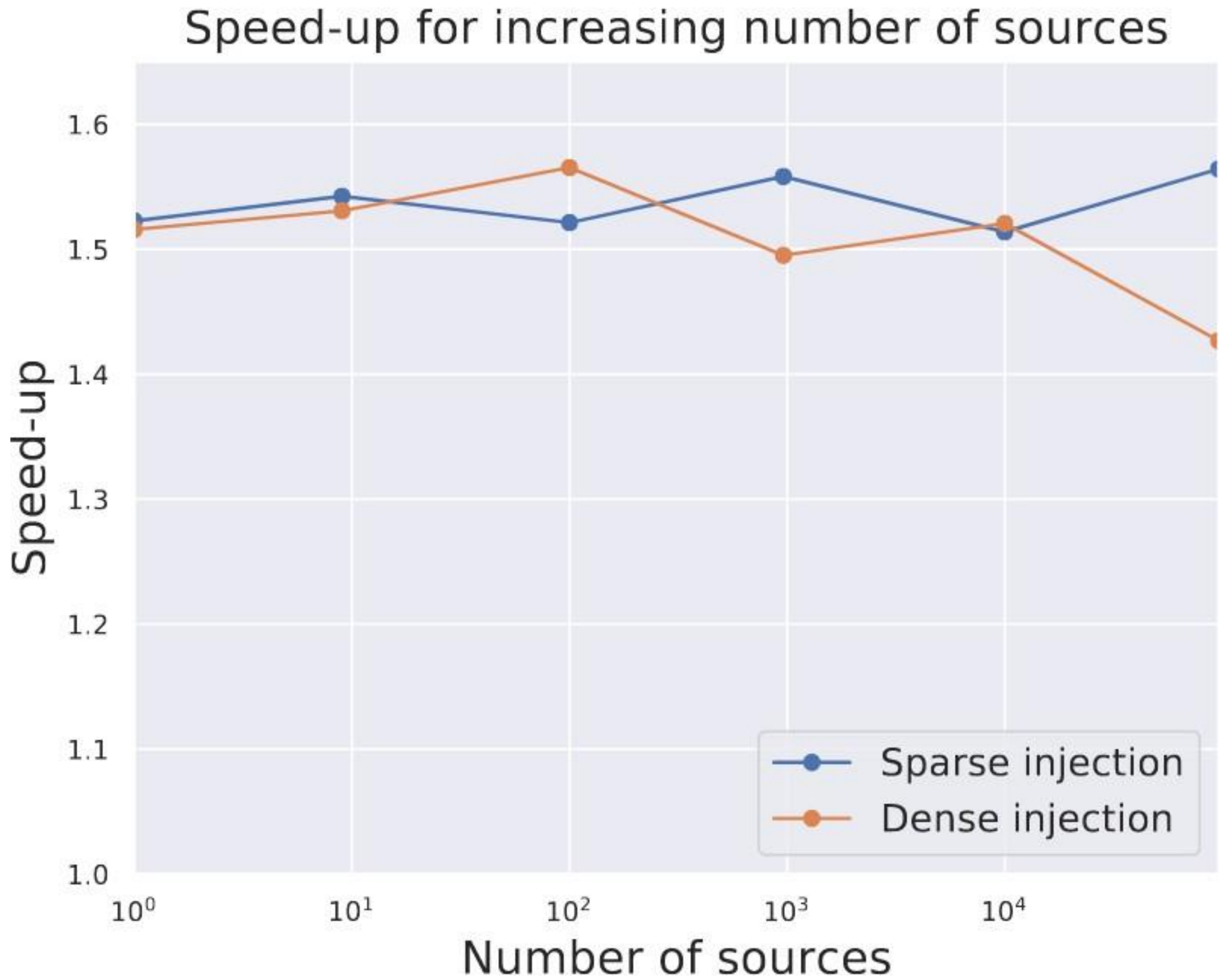
- *Bisbas, G., Luporini, F., Louboutin, M., Nelson, R., Gorman, G., & Kelly, P.H. (2020). Temporal blocking of finite-difference stencil operators with sparse "off-the-grid" sources. Accepted at IPDPS'21. Available online: https://arxiv.org/abs/2010.10248*

- Luporini, F., Lange, M., Louboutin, M., Kukreja, N., Hückelheim, J., Yount, C., Witte, P.A., Kelly, P.H., Gorman, G., & Herrmann, F. (2020). Architecture and Performance of Devito, a System for Automated Stencil Computation. ACM Transactions on Mathematical Software (TOMS), 46, 1 - 28.

- Louboutin, M., M., Lange, F., Luporini, N., Kukreja, P. A., Witte, F. J., Herrmann, P., Velesko, and G. J., Gorman. "Devito (v3.1.0): an embedded domain-specific language for finite differences and geophysical exploration".Geoscientific Model Development 12, no.3 (2019): 1165–1187.

- Yount, C., & Duran, A. (2016). Effective Use of Large High-Bandwidth Memory Caches in HPC Stencil Computation via Temporal Wave-Front Tiling. (2016) 7th International Workshop on Performance Modeling, Benchmarking and Simulation of High Performance Computer Systems (PMBS), 65-75.

Corner cases, increasing number of sources



The generated C code - stencil update

```c
    #pragma omp for collapse(1) schedule(dynamic,1)
    for (int x0_blk0 = x_m; x0_blk0 <= x_M; x0_blk0 += x0_blk0_size)
    {
      for (int y0_blk0 = y_m; y0_blk0 <= y_M; y0_blk0 += y0_blk0_size)
      {
        for (int x = x0_blk0; x <= x0_blk0 + x0_blk0_size - 1; x += 1)
        {
          for (int y = y0_blk0; y <= y0_blk0 + y0_blk0_size - 1; y += 1)
          {
            #pragma omp simd aligned(damp,uref,vp:32)
            for (int z = z_m; z <= z_M; z += 1)
            {
              float r14 = -2.84722222F*uref[t1][x + 8][y + 8][z + 8];
              float r13 = 1.0/dt;
              float r12 = 1.0/(dt*dt);
              float r11 = 1.0/(vp[x + 8][y + 8][z + 8]*vp[x + 8][y + 8][z + 8]);
              uref[t0][x + 8][y + 8][z + 8] = (r11*(-r12*(-2.0F*uref[t1][x + 8][y + 8][z + 8] +
uref[t2][x + 8][y + 8][z + 8])) + r13*(damp[x + 1][y + 1][z + 1]*uref[t1][x + 8][y + 8][z + 8])
(r14 - 1.78571429e-3F*(uref[t1][x + 8][y + 8][z + 4] + uref[t1][x + 8][y + 8][z + 12]) +
2.53968254e-2F*(uref[t1][x + 8][y + 8][z + 5] + uref[t1][x + 8][y + 8][z + 11]) -
2.0e-1F*(uref[t1][x + 8][y + 8][z + 6] + uref[t1][x + 8][y + 8][z + 10]) + 1.6F*(uref[t1][x + 8]
[y + 8][z + 7] + uref[t1][x + 8][y + 8][z + 9]))/((h_z*h_z)) + (r14 - 1.78571429e-3F*(uref[t1][x
+ 8][y + 4][z + 8] + uref[t1][x + 8][y + 12][z + 8]) + 2.53968254e-2F*(uref[t1][x + 8][y + 5][z
8] + uref[t1][x + 8][y + 11][z + 8]) - 2.0e-1F*(uref[t1][x + 8][y + 6][z + 8] + uref[t1][x + 8][
+ 10][z + 8]) + 1.6F*(uref[t1][x + 8][y + 7][z + 8] + uref[t1][x + 8][y + 9][z + 8]))/((h_y*h_y)
+ (r14 - 1.78571429e-3F*(uref[t1][x + 4][y + 8][z + 8] + uref[t1][x + 12][y + 8][z + 8]) +
2.53968254e-2F*(uref[t1][x + 5][y + 8][z + 8] + uref[t1][x + 11][y + 8][z + 8]) -
2.0e-1F*(uref[t1][x + 6][y + 8][z + 8] + uref[t1][x + 10][y + 8][z + 8]) + 1.6F*(uref[t1][x + 7]
[y + 8][z + 8] + uref[t1][x + 9][y + 8][z + 8]))/((h_x*h_x)))/(r11*r12 + r13*damp[x + 1][y + 1][
+ 1]);
            }
          }
        }
      }
    }
  }
}
```

The generated C code - source injection

```
/* Begin section1 */
#pragma omp parallel num_threads(nthreads_nonaffine)
{
  int chunk_size = (int)(fmax(1, (1.0F/3.0F)*(p_src_M - p_src_m + 1)/nthreads_nonaffine));
  #pragma omp for collapse(1) schedule(dynamic,chunk_size)
  for (int p_src = p_src_m; p_src <= p_src_M; p_src += 1)
  {
    int ii_src_0 = (int)(floor((-o_x + src_coords[p_src][0])/h_x));
    int ii_src_1 = (int)(floor((-o_y + src_coords[p_src][1])/h_y));
    int ii_src_2 = (int)(floor((-o_z + src_coords[p_src][2])/h_z));
    int ii_src_3 = (int)(floor((-o_z + src_coords[p_src][2])/h_z)) + 1;
    int ii_src_4 = (int)(floor((-o_y + src_coords[p_src][1])/h_y)) + 1;
    int ii_src_5 = (int)(floor((-o_x + src_coords[p_src][0])/h_x)) + 1;
    float px = (float)(-h_x*(int)(floor((-o_x + src_coords[p_src][0])/h_x)) - o_x + src_coords[p_src][0]);
    float py = (float)(-h_y*(int)(floor((-o_y + src_coords[p_src][1])/h_y)) - o_y + src_coords[p_src][1]);
    float pz = (float)(-h_z*(int)(floor((-o_z + src_coords[p_src][2])/h_z)) - o_z + src_coords[p_src][2]);
    if (ii_src_0 >= x_m - 1 && ii_src_1 >= y_m - 1 && ii_src_2 >= z_m - 1 && ii_src_0 <= x_M + 1 && ii_src_1
<= y_M + 1 && ii_src_2 <= z_M + 1)
    {
      float r0 = 4.49016082216644F*(vp[ii_src_0 + 8][ii_src_1 + 8][ii_src_2 + 8]*vp[ii_src_0 + 8][ii_src_1 + 8]
[ii_src_2 + 8])*(-px*py*pz/(h_x*h_y*h_z) + px*py/(h_x*h_y) + px*pz/(h_x*h_z) - px/h_x + py*pz/(h_y*h_z) - py/h_y -
pz/h_z + 1)*src[time][p_src];
      #pragma omp atomic update
      uref[t0][ii_src_0 + 8][ii_src_1 + 8][ii_src_2 + 8] += r0;
    }
    if (ii_src_0 >= x_m - 1 && ii_src_1 >= y_m - 1 && ii_src_3 >= z_m - 1 && ii_src_0 <= x_M + 1 && ii_src_1
<= y_M + 1 && ii_src_3 <= z_M + 1)
    {
      float r1 = 4.49016082216644F*(vp[ii_src_0 + 8][ii_src_1 + 8][ii_src_3 + 8]*vp[ii_src_0 + 8][ii_src_1 + 8]
[ii_src_3 + 8])*(px*py*pz/(h_x*h_y*h_z) - px*pz/(h_x*h_z) - py*pz/(h_y*h_z) + pz/h_z)*src[time][p_src];
      #pragma omp atomic update
      uref[t0][ii_src_0 + 8][ii_src_1 + 8][ii_src_3 + 8] += r1;
    }
    if (ii_src_0 >= x_m - 1 && ii_src_2 >= z_m - 1 && ii_src_4 >= y_m - 1 && ii_src_0 <= x_M + 1 && ii_src_2
<= z_M + 1 && ii_src_4 <= y_M + 1)
    {
      float r2 = 4.49016082216644F*(vp[ii_src_0 + 8][ii_src_4 + 8][ii_src_2 + 8]*vp[ii_src_0 + 8][ii_src_4 + 8]
[ii_src_2 + 8])*(px*py*pz/(h_x*h_y*h_z) - px*py/(h_x*h_y) - py*pz/(h_y*h_z) + py/h_y)*src[time][p_src];
```

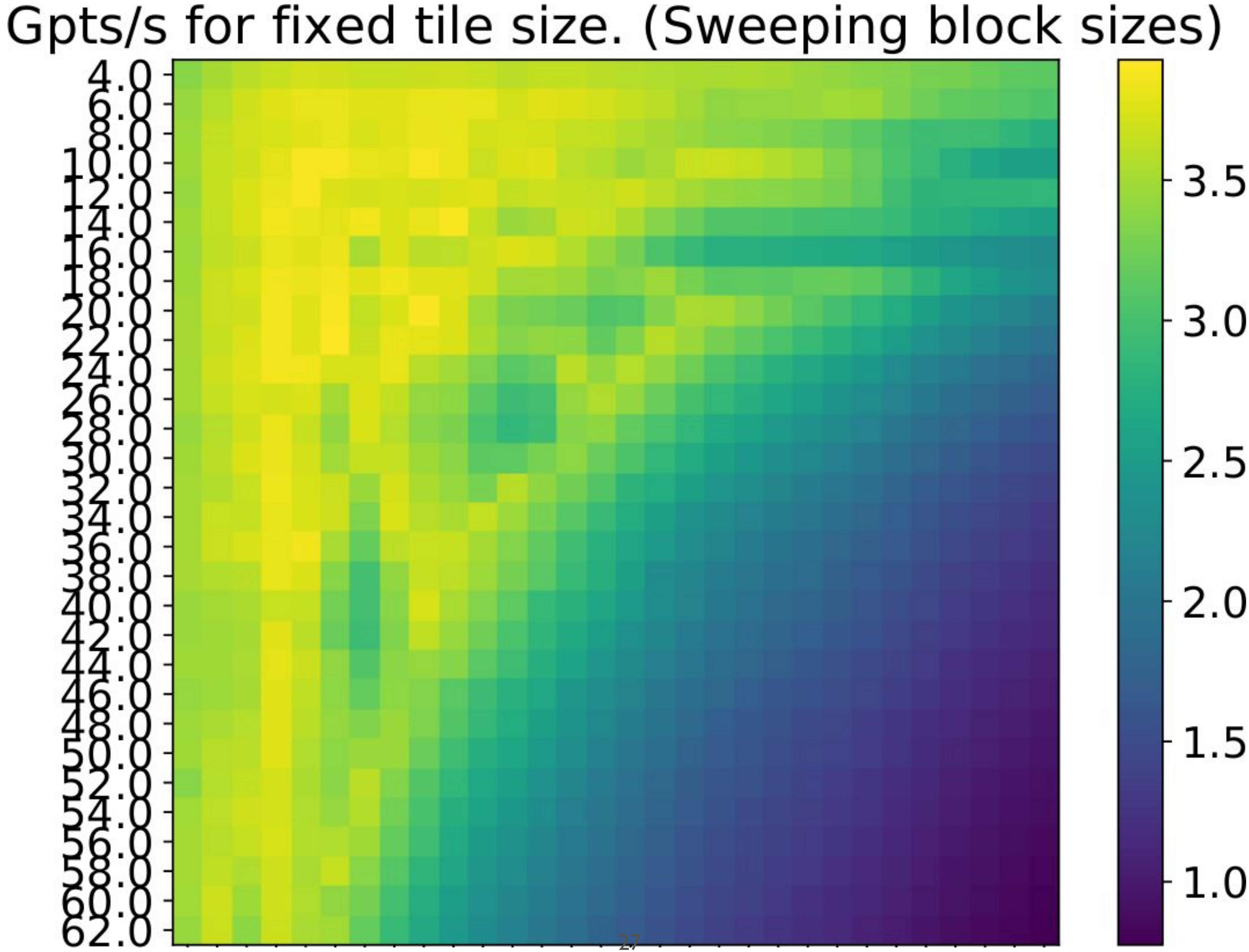
Algorithm 3: Source injection pseudocode.

```
for t = 1 to nt do
    foreach s in sources do
        # Find on the grid coordinates
        src_x_min = floor(src_coords[s][0], ox)
        src_x_max = ceil(src_coords[s][0], ox)
        src_y_min = floor(src_coords[s][1], oy)
        src_y_max = ceil(src_coords[s][1], oy)
        src_z_min = floor(src_coords[s][2], oz)
        src_z_max = ceil(src_coords[s][2], oz)
        # Compute weights
        px = f(src_coords[s][0], ox)
        py = f(src_coords[s][1], oy)
        pz = f(src_coords[s][2], oz)
        # Unrolled for 8 points
        if src_x_min, src_y_min, src_z_min in grid then
            r0 = v(src_x_min, src_y_min, src_z_min, src[t][s]);
            u[t, src_x_min, src_y_min, src_z_min] + = r0)
        .
        .
        .
        if src_x_max, src_y_max, src_z_max in grid then
            r7 = v(src_x_max, src_y_max, src_z_max src[t][s]);
            u[t, src_x_max, src_y_max, src_z_max] + = r7)
```

Floor, ceil of off- the-grid

Weights of impact

Unrolled loop for each point, compute injection part affected and add to field a


Gpts/s for fixed tile size. (Sweeping block sizes)
4.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
20.0
22.0
24.0
26.0
28.0
30.0
32.0
34.0
36.0
38.0
40.0
42.0
44.0
46.0
48.0
50.0
52.0
54.0
56.0
58.0
60.0
62.0
3.5
3.0
2.5
2.0
1.5
1.0

Algorithm 3: Source injection pseudocode.

```
for t = 1 to nt do
    foreach s in sources do
        # Find on the grid coordinates
        src_x_min = floor(src_coords[s][0], ox)
        src_x_max = ceil(src_coords[s][0], ox)
        ⋮
        # Compute weights
        px = f(src_coords[s][0], ox)
        ⋮
        # Unrolled for 8 points (2^3, 3D case)
        if src_x_min, ... in grid then
            r0 = v(src_x_min, ...src[t][s]);
            u[t, src_x_min, ...] + = r0)
        ⋮
        if src_x_max, ... in grid then
            r7 = v(src_x_max, ...src[t][s]);
            u[t, src_x_max, ...] + = r7)
```

Cache aware roofline model

From here: https://crd.lbl.gov/departments/computer-science/par/research/roofline/introduction/

Effects of Cache Behavior on Arithmetic Intensity
The Roofline model requires an estimate of total data movement. On cache-based architectures, the 3C's cache model highlights the fact that there can be more than simply compulsory data movement. Cache capacity and conflict misses can increase data movement and reduce arithmetic intensity.
Similarly, superfluous cache write-allocations can result in a doubling of data movement. The vector initialization operation x[i]=0.0 demands one write allocate and one write back per cache line touched. The write allocate is superfluous as all elements of that cache line are to be overwritten. Unfortunately, the presence of hardware stream prefetchers can make it very difficult to quantify how much beyond compulsory data movement actually occurred.

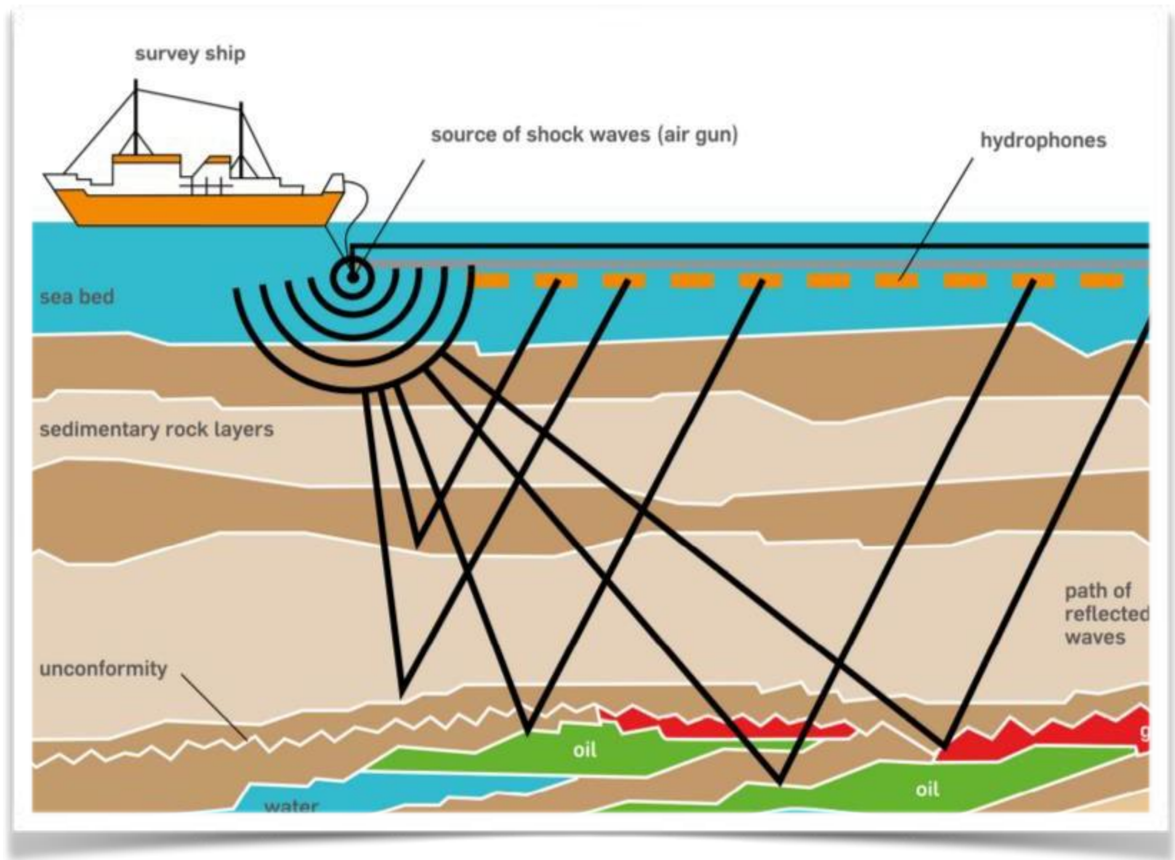
A bit of background

- **PDEs** are everywhere:
  computational fluid dynamics, image processing, weather forecasting, seismic and medical imaging.

- Numerical analysis => **finite-difference (FD)** methods to solve DEs by approximating derivatives with finite differences.

- **Devito:** Fast Stencil Computation from Symbolic Specification

- **Goal**:
  To improve performance of stencils stemming from practical applications using temporal blocking



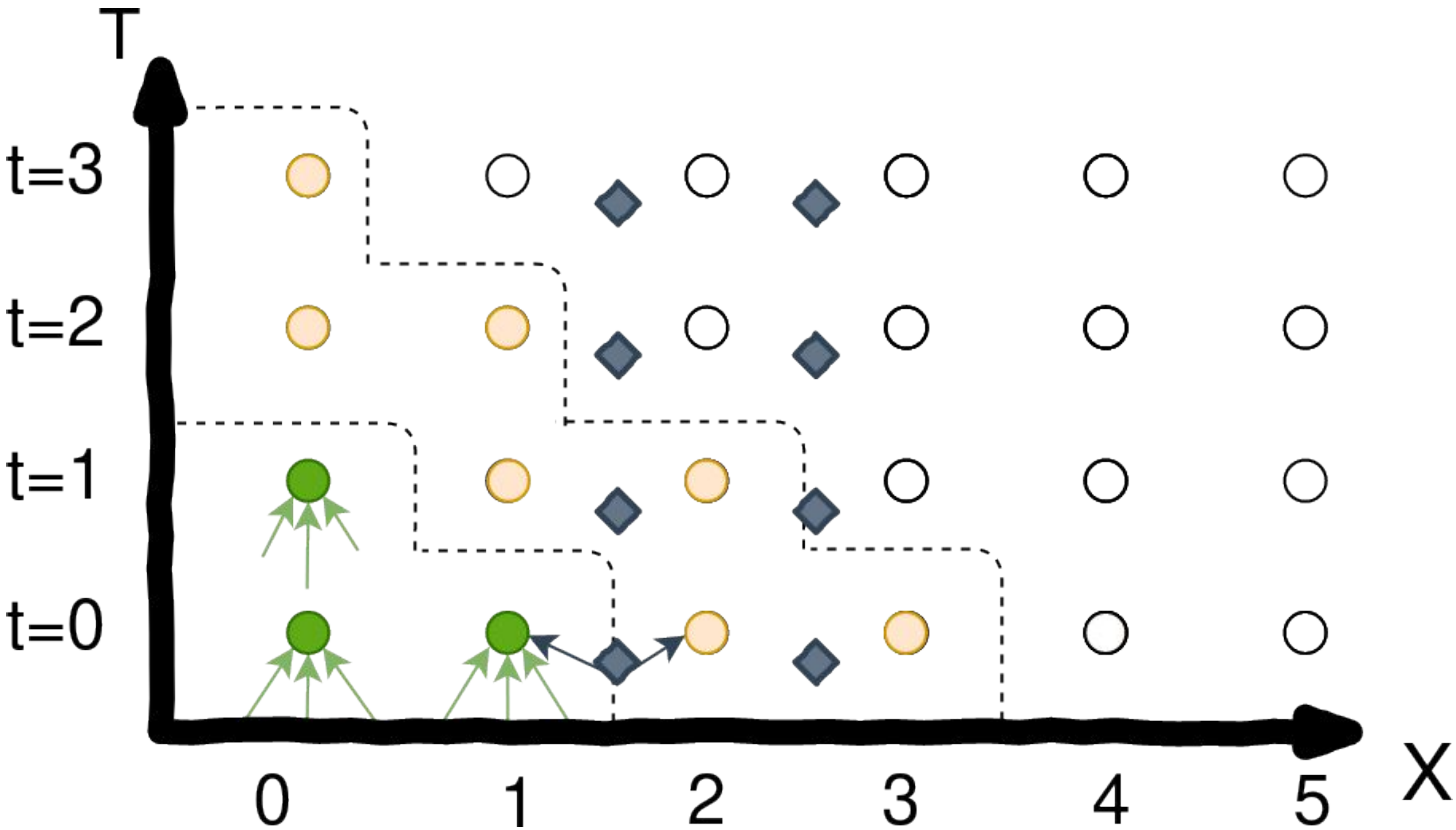
Algorithm 3: Source injection pseudocode.

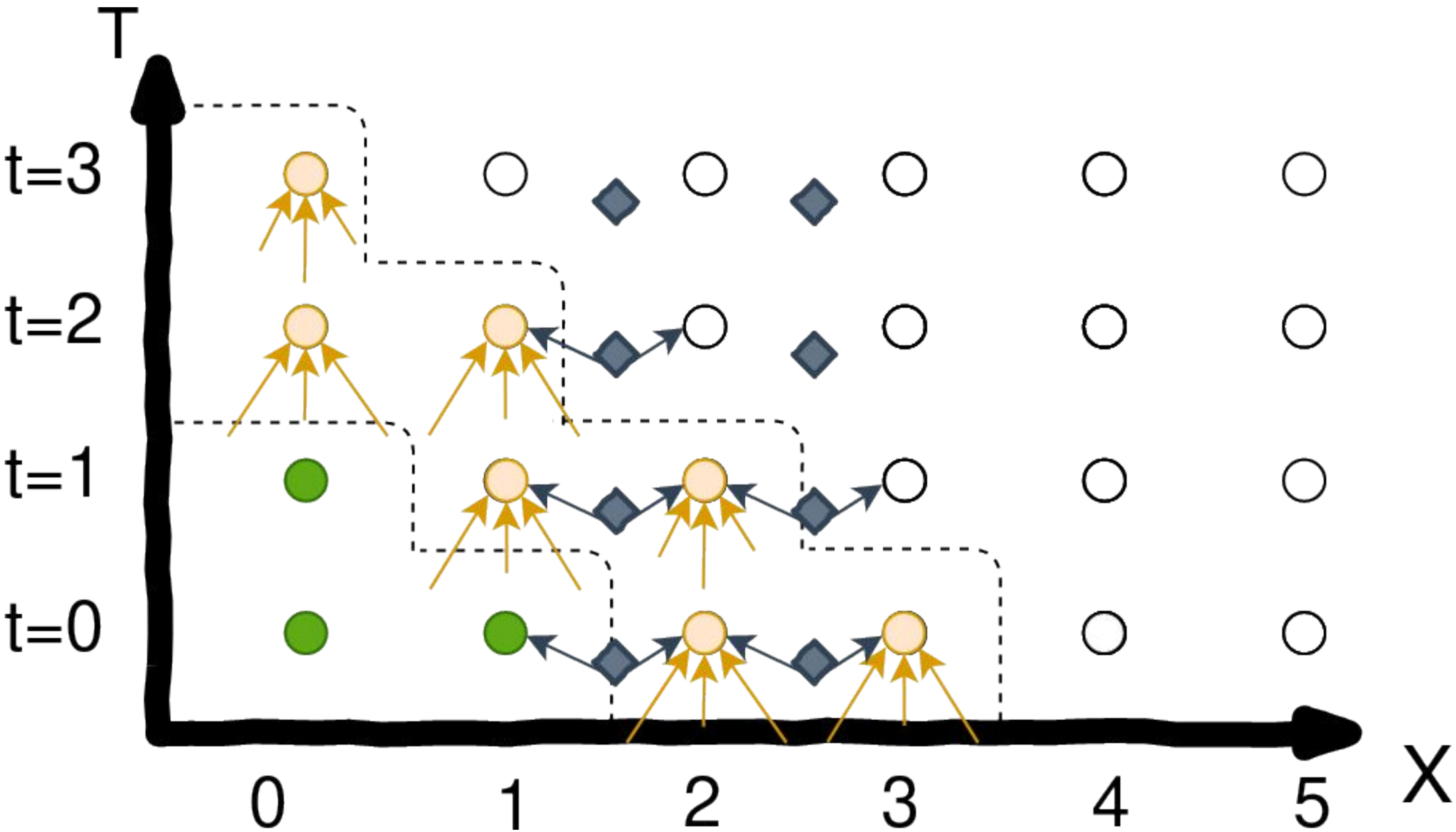
```
for t = 1 to nt do
    foreach s in sources do
        # Find on the grid coordinates
        src_x_min = floor(src_coords[s][0], ox)
        src_x_max = ceil(src_coords[s][0], ox)
        ⋮
        # Compute weights
        px = f(src_coords[s][0], ox)
        ⋮
        # Unrolled for 8 points (2^3, 3D case)
        if src_x_min, ... in grid then
            r0 = v(src_x_min, ...src[t][s]);
            u[t, src_x_min, ...] + = r0)
        ⋮
        if src_x_max, ... in grid then
            r7 = v(src_x_max, ...src[t][s]);
            u[t, src_x_max, ...] + = r7)
```

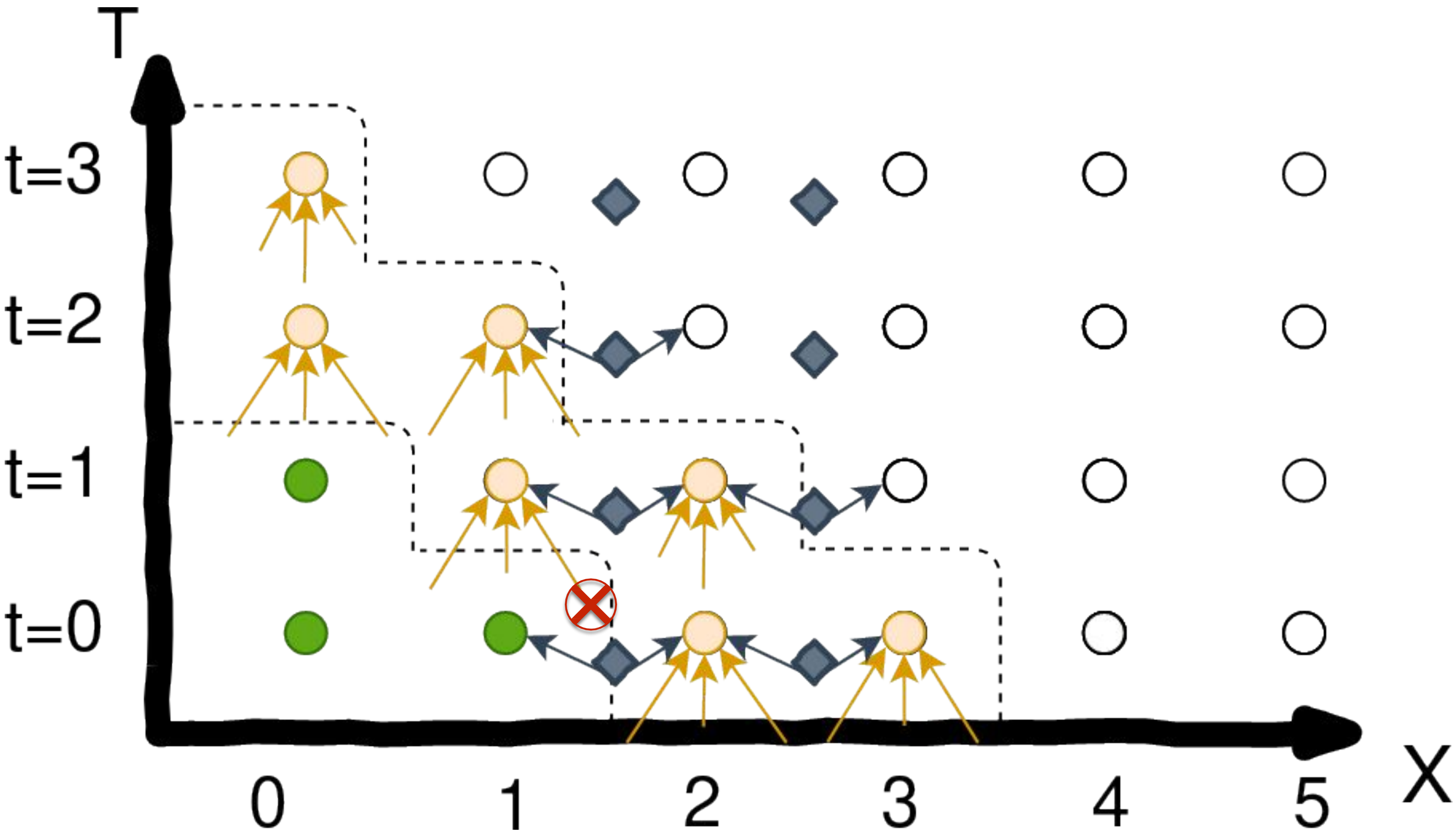
Discover affected points

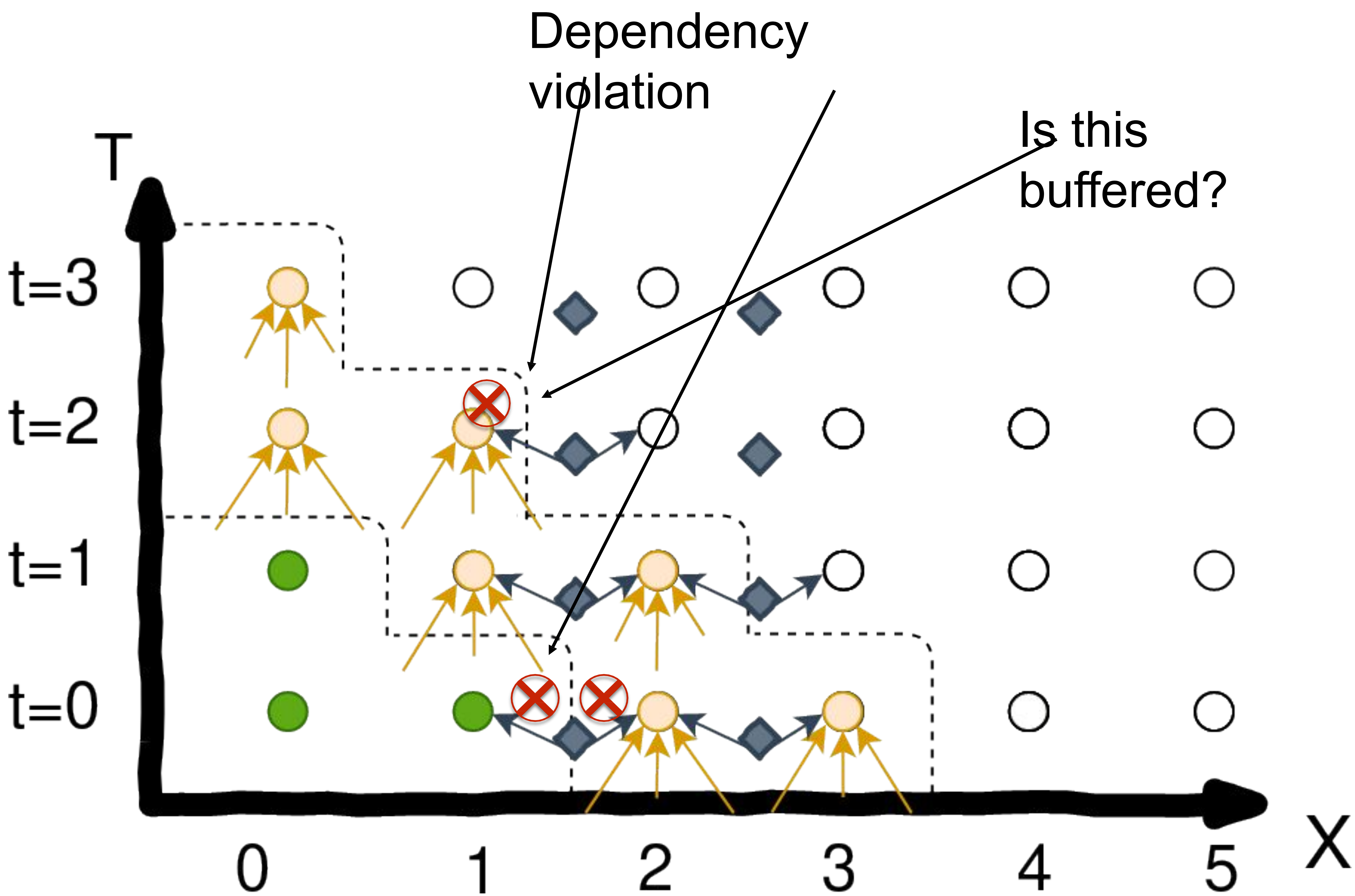
Weights of impact

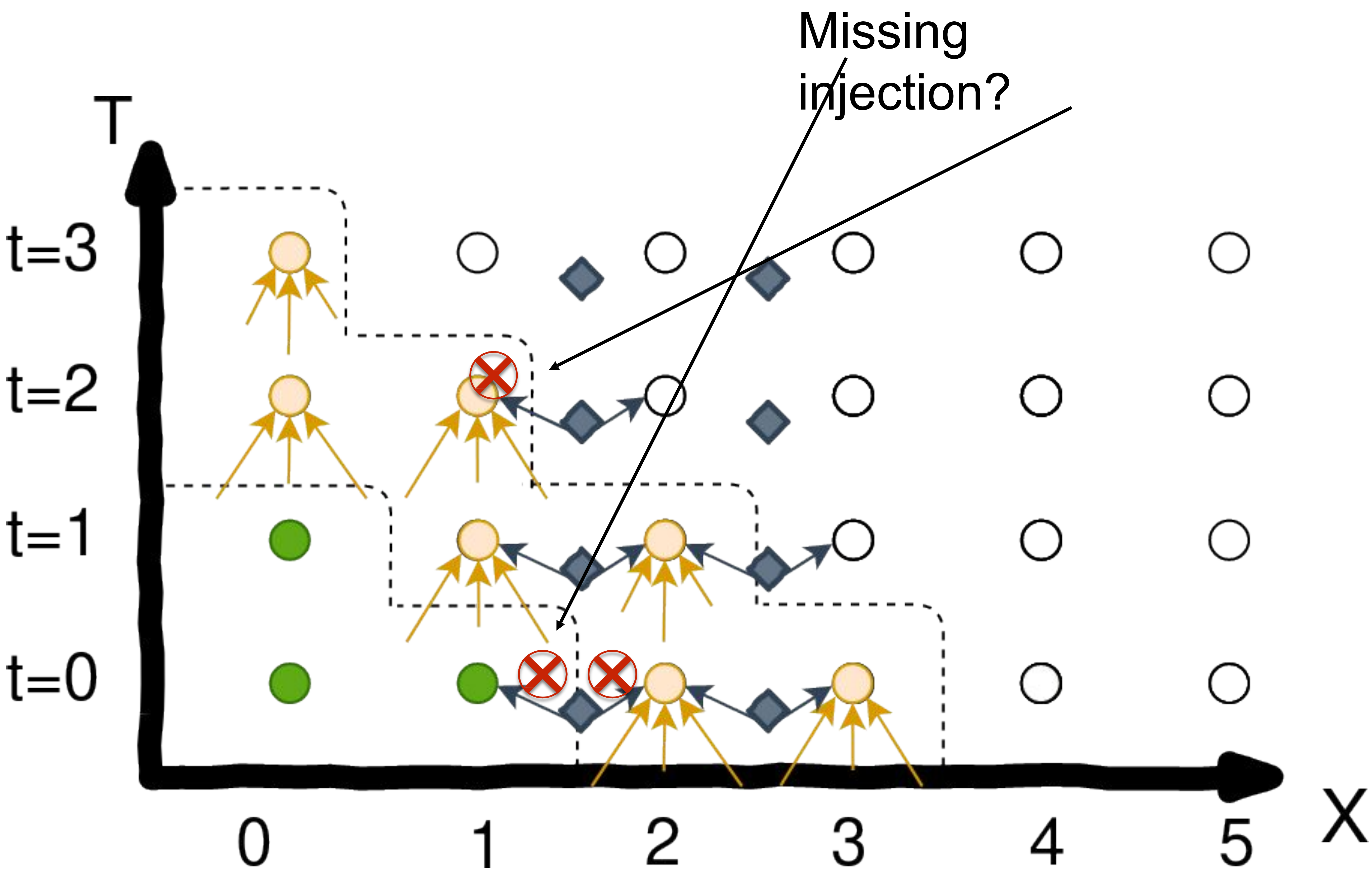
Unrolled loop for each affected point, compute injection part and add to field

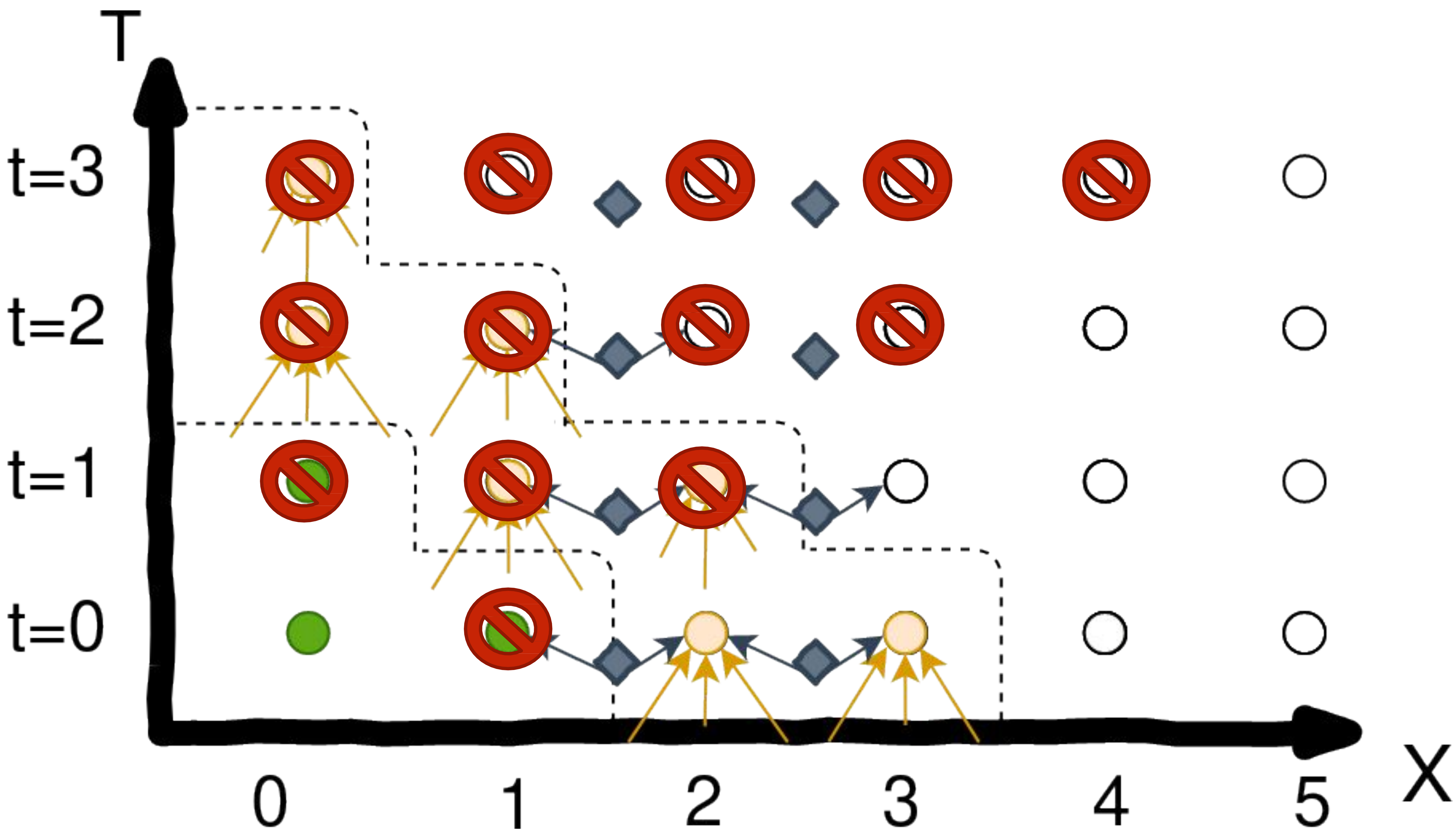

T
t=3
t=2
t=1
t=0
0
1
2
3
4
5
X


T
t=3
t=2
t=1
t=0
0
1
2
3
4
5
X


T
t=3
t=2
t=1
t=0
0
1
2
3
4
5
X


Dependency violation
Is this buffered?
T
t=3
t=2
t=1
t=0
0
1
2
3
4
5
X


Missing injection?
T
t=3
t=2
t=1
t=0
0
1
2
3
4
5
X


T
t=3
t=2
t=1
t=0
0
1
2
3
4
5
X

Algorithm 1: A typical time-stepping loop nest structure for a stencil update with source injection. This stencil has one temporal and three spatial dimensions.

for t = 1 ***to*** nt **do**
  for x = 1 ***to*** nx **do**
    for y = 1 ***to*** ny **do**
      for z = 1 ***to*** nz **do**
        $A(t, x, y, z) \equiv$ u[t, x, y, z] = u[t-1, x, y, z] + $\sum_{r=1}^{r=so/2} w_r \Big[$ u[t-1, x - r, y, z] + u[t-1, x + r, y, z] + u[t-1, x, y - r, z] + u[t-1, x, y + r, z] + u[t-1, x, y, z - r] + u[t-1, x, y, z + r] $\Big]$;
  foreach s ***in*** sources **do**
    for i = 1 ***to*** np **do**
      xs, ys, zs = map(s, i);
      u[t, xs, ys, zs] + = $f(src(t, s))$

Non-aligned

Algorithm 6: Stencil kernel update with fused - reduced size iteration space - source injection.

for t = 1 ***to*** nt **do**
  for x = 1 ***to*** nx **do**
    for y = 1 ***to*** ny **do**
      for z = 1 ***to*** nz **do**
        $A(t, x, y, z, s)$;
      for $z2$ = 1 ***to*** nnz_mask[x][y] **do**
        zind = Sp_SM[x, y, z];
        u[t, x, y, z2] +=
        SM[x, y, zind] * src_dcmp[t, SID[x, y, zind]];

Aligne
d

Listing 1: A typical time-stepping loop nest structure for a stencil update with source injection. This stencil has one temporal and three spatial dimensions.

```
for t = 1 to nt do
    for x = 1 to nx do
        for y = 1 to ny do
            for z = 1 to nz do
                A(t, x, y, z) ≡ u[t, x, y, z] = u[t-1, x, y, z] + Σ_{r=1}^{r=so/2} w_r (
                  u[t-1, x - r, y, z] + u[t-1, x + r, y, z] + u[t-1, x, y - r, z] +
                  u[t-1, x, y + r, z] + u[t-1, x, y, z - r] + u[t-1, x, y, z + r] );
    foreach s in sources do // For every source
        for i = 1 to np do // Get the points affected
            xs, ys, zs = map(s, i) // through indirection
            u[t, xs, ys, zs] + = f(src(t, s)) // add their impact
              on the field
```

Non-aligned

Aligned to grid Same OPS Parallelism

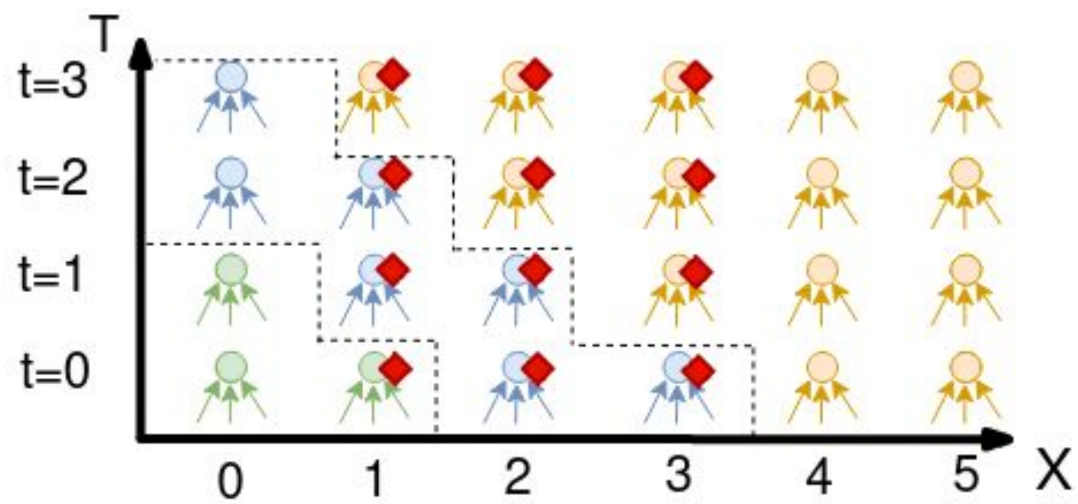
SIMD (?)

Apply TB

Listing 5: Stencil kernel update with fused - reduced size iteration space - source injection.

```
for t = 1 to nt do
    for x = 1 to nx do
        for y = 1 to ny do
            for z = 1 to nz do
                A(t, x, y, z, s);
            for z2 = 1 to nnz_mask[x][y] do
                I(t, x, y, z, s) ≡{ zind = Sp_SM[x, y, z2];
                u[t, x, y, z2] +=
                SM[x, y, zind] * src_dcmp[t, SID[x, y, zind]]; }
```

Aligned



The transformation in Devito-DSL

```
u = TimeFunction(name="u", grid=model.grid, space_order=so, time_order=2)
src_term = src.inject(field=u.forward, expr=src * dt**2 / model.m)
pde = model.m * u.dt2 - u.laplace + model.damp * u.dt
stencil = Eq(u.forward, solve(pde, u.forward))
op = Operator([stencil, src_term])
```

The transformation in Devito-DSL

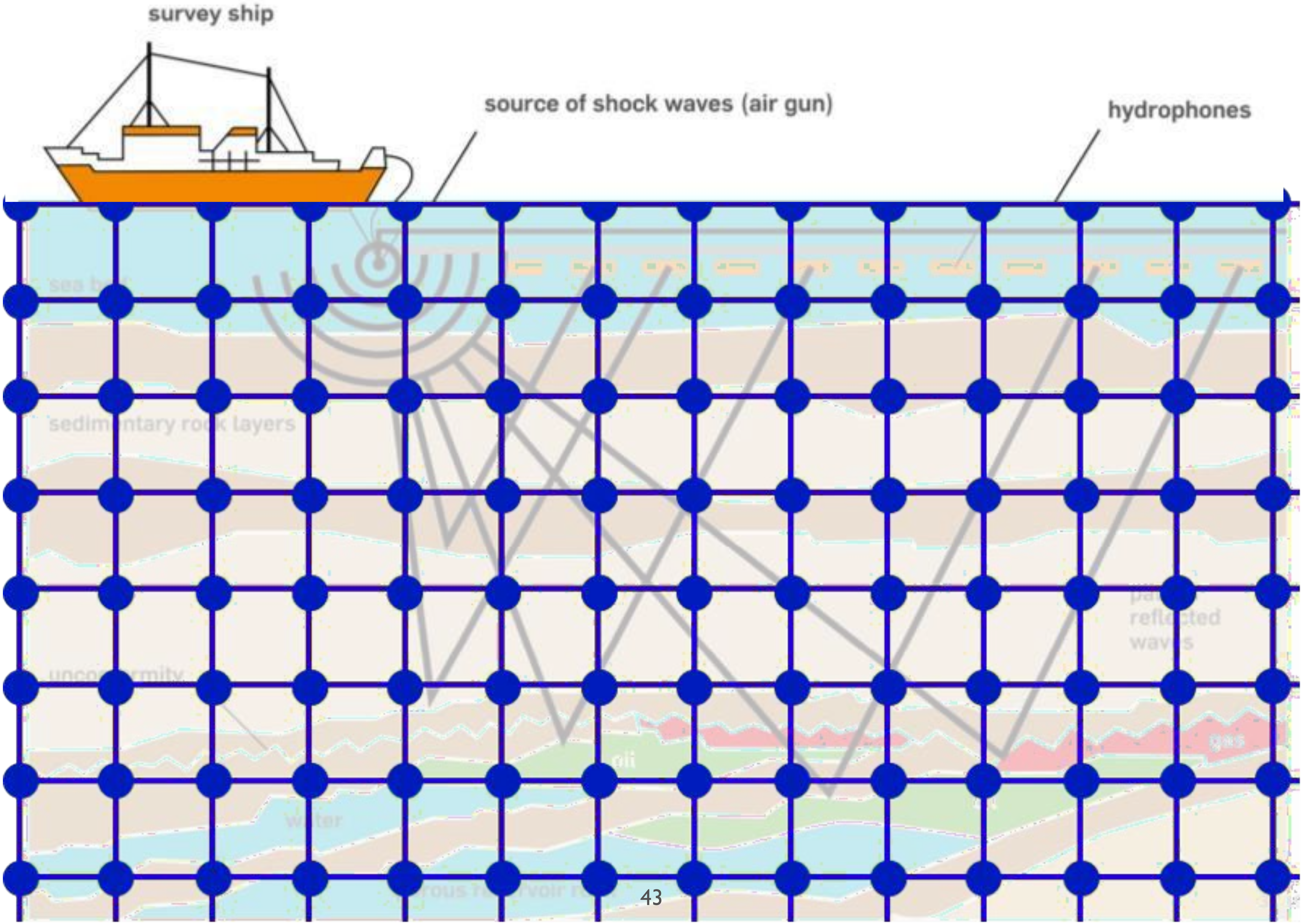
```
# f : perform source injection on an empty grid
f = TimeFunction(name="f", grid=model.grid, space_order=so, time_order=2)
src_f = src.inject(field=f.forward, expr=src * dt**2 / model.m)
op_f = Operator([src_f])
op_f_sum = op_f.apply(time=3)

nzinds = np.nonzero(f.data[0]) # nzinds is a tuple
.
.
.
eq0 = Eq(sp_zi.symbolic_max, nnz_sp_source_mask[x, y] - 1, implicit_dims=(time, x, y))
eq1 = Eq(zind, sp_source_mask[x, y, sp_zi], implicit_dims=(time, x, y, sp_zi))

mask_expr = source_mask[x, y, zind] * save_src[time, source_id[x, y, zind]]
eq2 = Inc(usol.forward[t+1, x, y, zind], mask_expr, implicit_dims=(time, x, y, sp_zi))

pde_2 = model.m * usol.dt2 - usol.laplace + model.damp * usol.dt

stencil_2 = Eq(usol.forward, solve(pde_2, usol.forward))
```

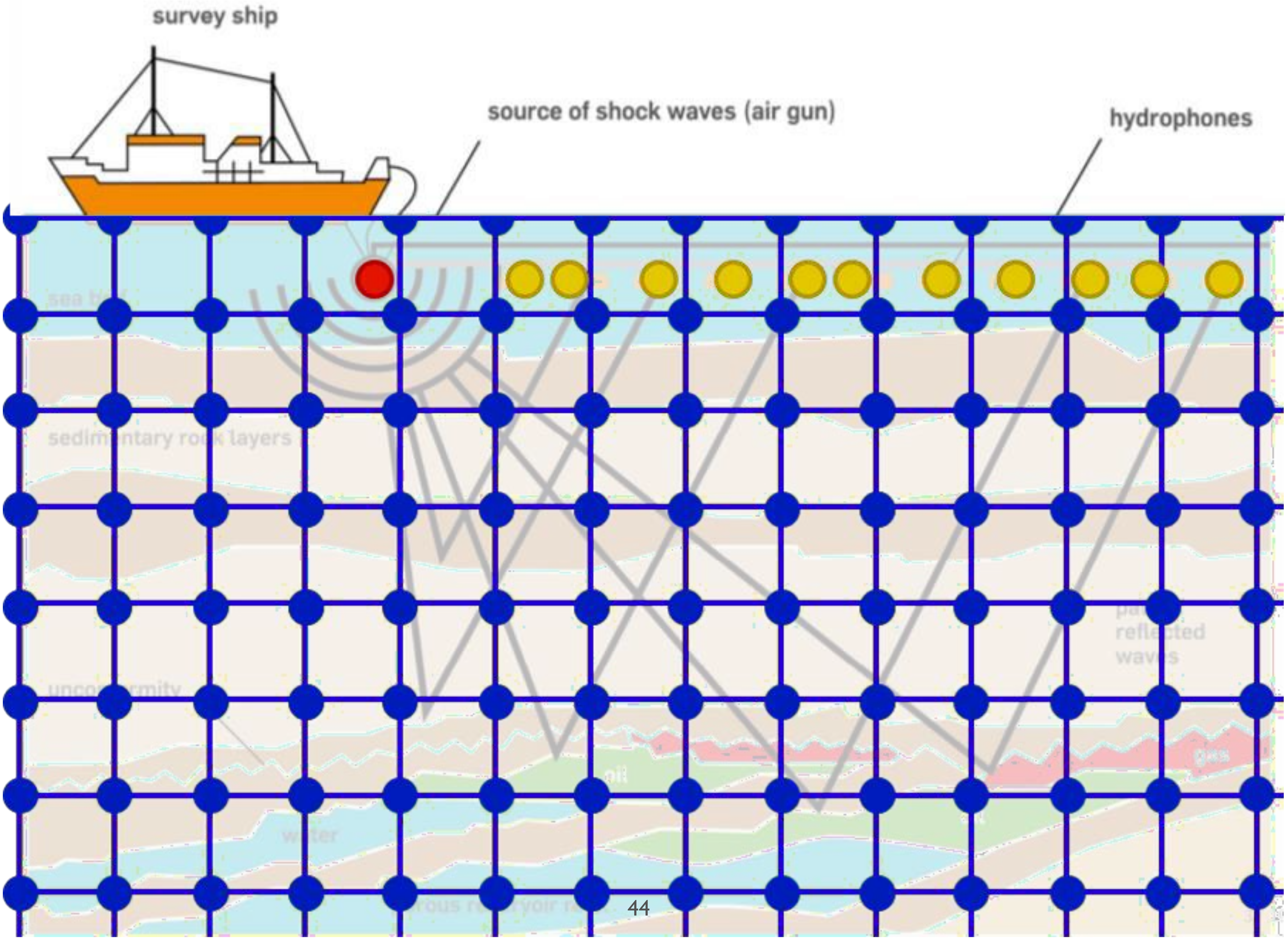
Fuse iteration spaces

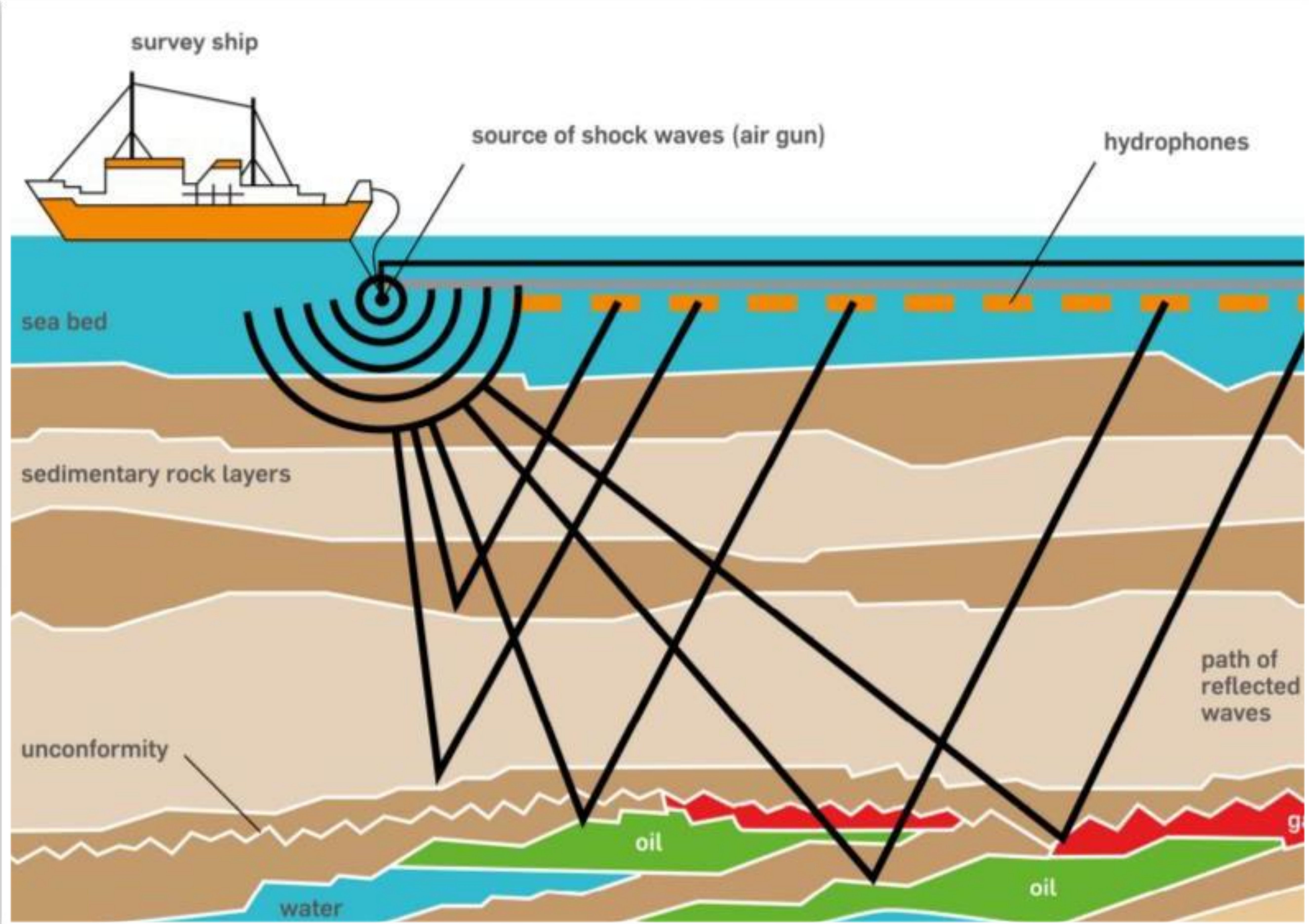
- Indirection mapping has changed. We still use indirections but now they are on the point.
- By using the aligned structure, we fuse the source injection loop inside the kernel update iteration space.
- The source mask SM is used to add (if 1) or not (if 0) the impact and SID is used to indirect to the impact values using the traversed grid coordinates.

Listing 4: Stencil kernel update with fused source injection.

```
for t = 1 to nt do
    for x = 1 to nx do
        for y = 1 to ny do
            for z = 1 to nz do
                A(t, x, y, z, s);
            for z2 = 1 to nz do
                u[t, x, y, z2] += SM[x, y, z2] * src_dcmp[t, SID[x, y, z2]];
```

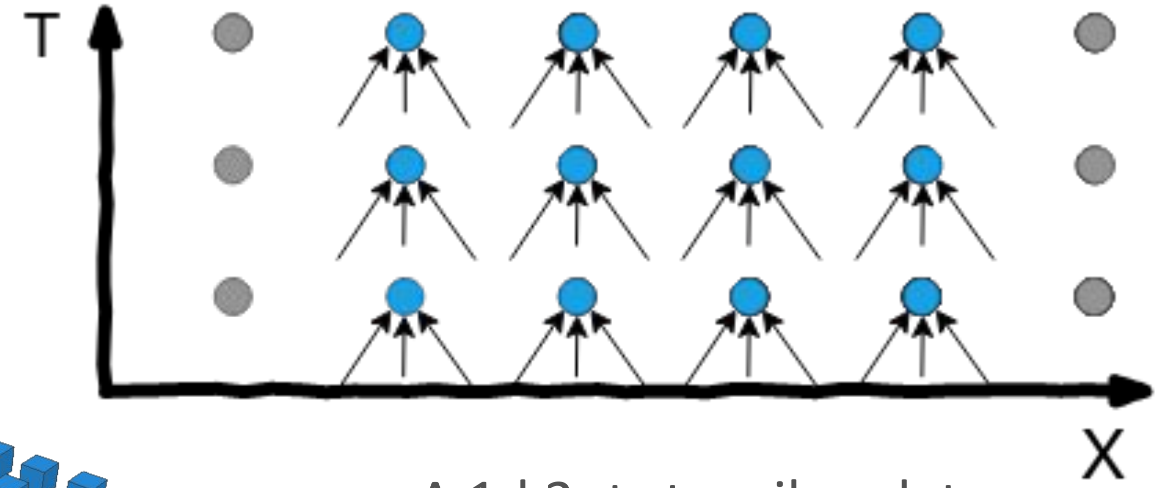

survey ship
source of shock waves (air gun)
hydrophones


survey ship
source of shock waves (air gun)
hydrophones


survey ship
source of shock waves (air gun)
hydrophones
sea bed
sedimentary rock layers
path of reflected waves
unconformity
oil
oil
water

Stencils are everywhere

- Vast literature on optimizing stencils…
- Parallelism, cache optimizations, accelerators
- From simplistic (1d-3pt)
  ...to wide and
  ...complex



A 1d 3pt stencil update

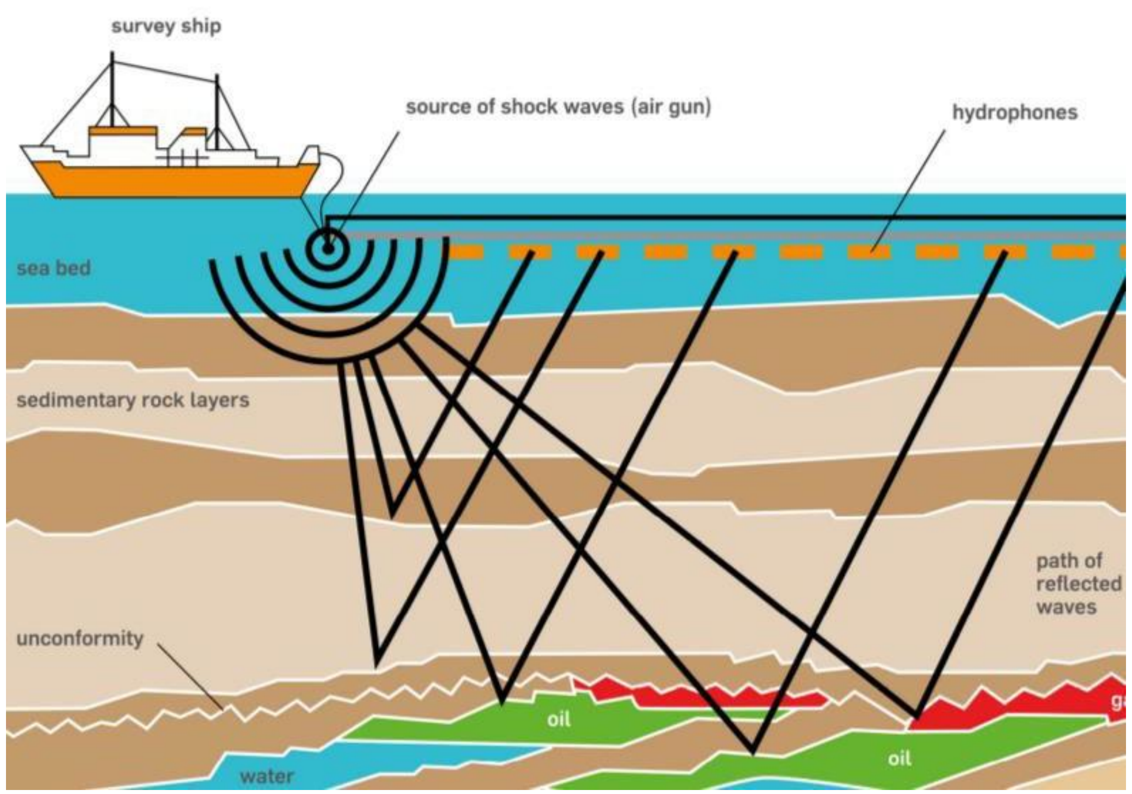
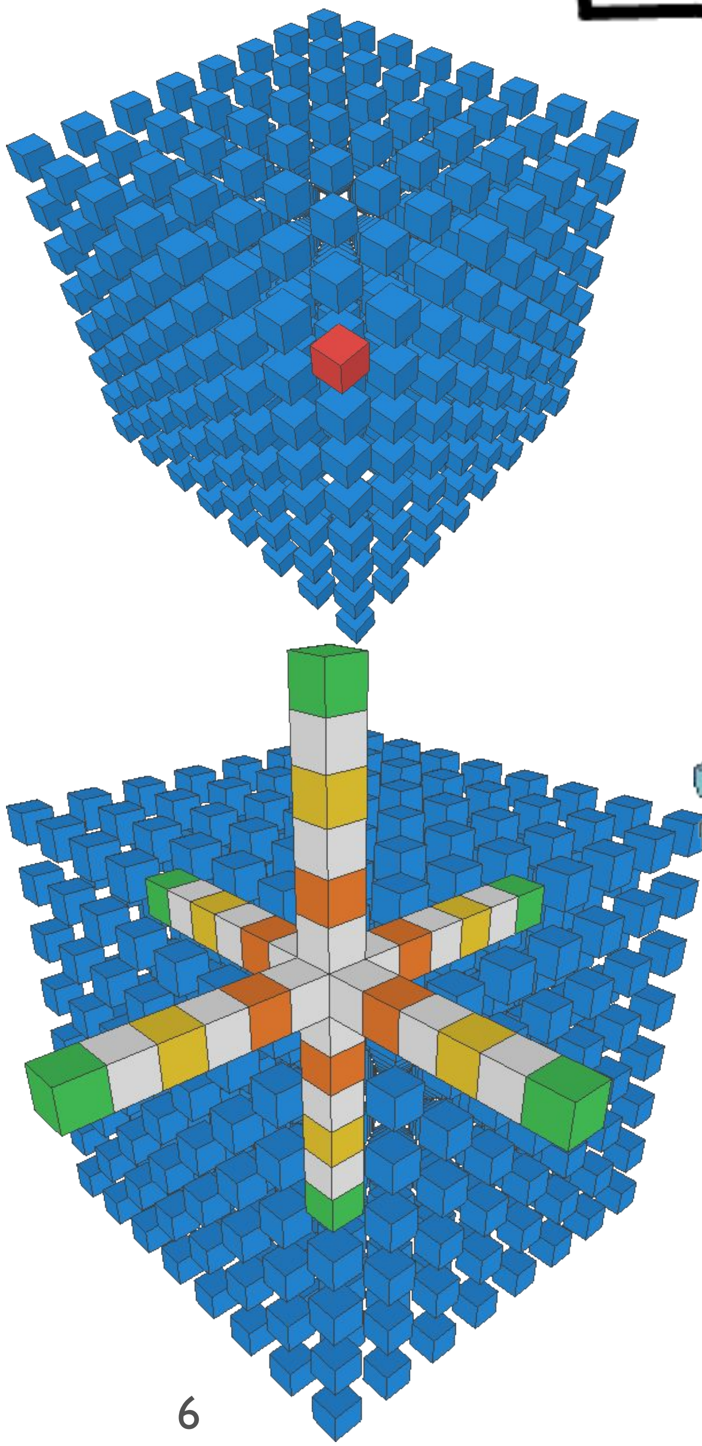
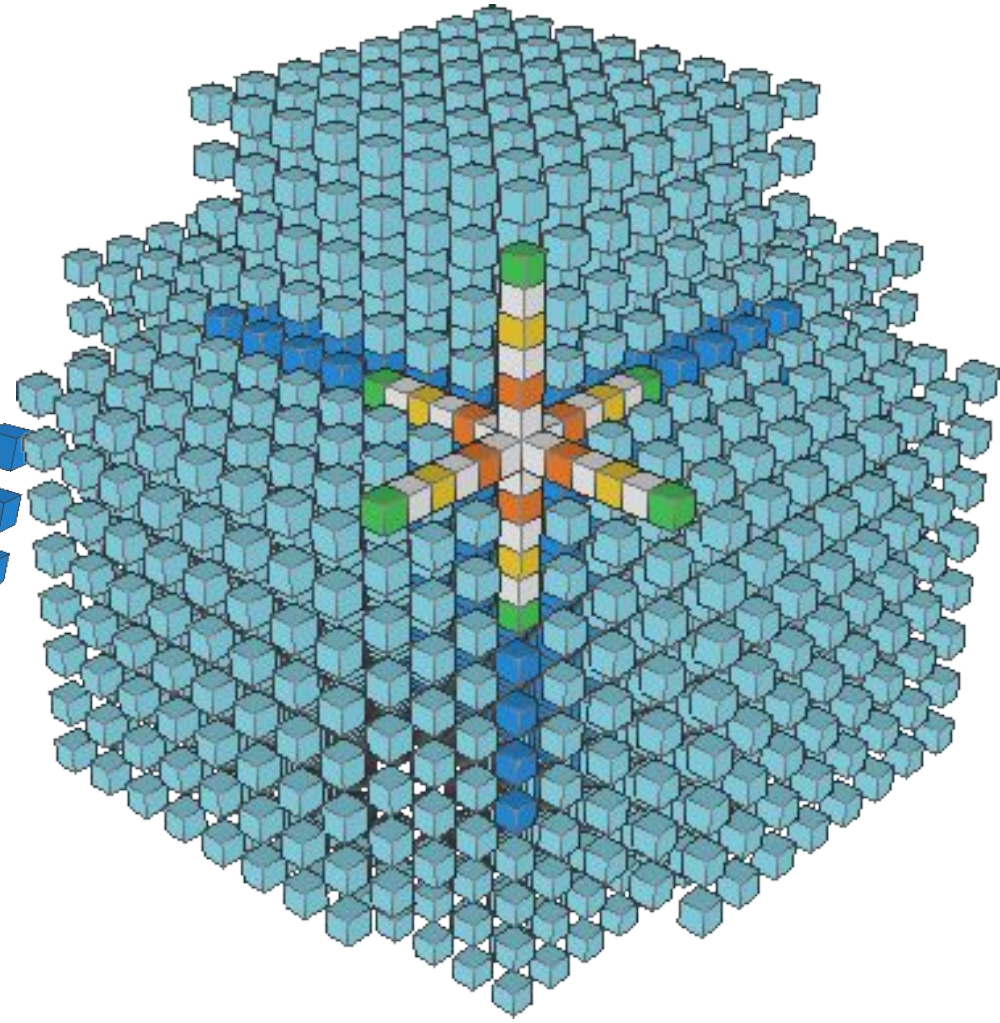


Illustration of an Offshore Seismic Survey, Source: Kris Energy





A 3d-19pt stencil update