Leveraging symbolic math for rapid development of applications for seismic imaging

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Introduction

Motivation

Example

Seismic Imaging - Motivation

Use geophysics to understand the earth

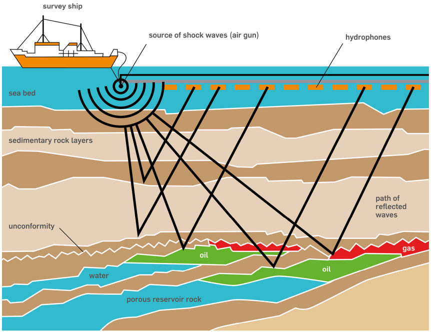
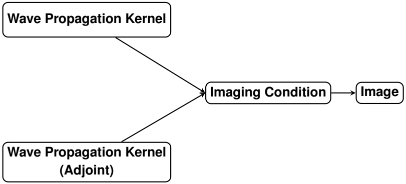
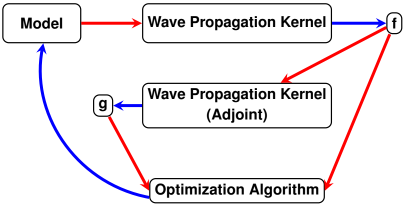


Figure 1: Offshore seismic survey

Source: http://www.open.edu/openlearn/science-maths-technology/science/environmental-science/earths-physical-resources-petroleum/content-section-3.2.1

Imaging and Inversion





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Why does it need to be fast?

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- Large number of operations: $\approx$ 6000 FLOPs per loop iteration of a 16th order TTI kernel
- Realistic problems have large grids: 1580 x 1580 x 1130 $\approx$ 2.82 billion points (SEAM benchmark [1])
- $2.82 \times 10^9 \times 6000 \times 3000(t) \times 2$ (forward-reverse) $\approx 10^{17}$ FLOPs per shot
- Typically $\approx$ 30000 shots ($\approx 3 \times 10^{21} = 3 \times 10^9$ TFLOPs per FWI iteration)
- Typically $\approx$ 15 FWI iterations ($\approx 4.6 \times 10^{22} =$ 46 billion TFLOPs total)

$\approx$ 100 wall-clock days executing on the TACC Stampede (assuming linpack-level performance)

[1] Michael Fehler and P. Joseph Keliher. *SEAM Phase I.* Society of Exploration Geophysicists, 2011

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Why automated

OPESCI

Computer science

- Fast code is complex
  - Loop blocking
  - OpenMP clauses
  - Vectorization - intrinsics
  - Memory - alignment, NUMA
  - Common sub-expression elimination
  - ADD/MUL balance
  - Denormal numbers
  - Elemental functions
  - Non temporal stores
- Fast code is platform dependent
  - Intrinsics
  - CUDA/OpenCL
  - Data layouts
- Fast code is error prone

Geophysics

- Change of discretizations - Numerical analysis
- Change of physics
  - Anisotropy - VTI/TTI
  - Elastic equation
- Boundary conditions

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Not everyone is a polymath

Devito - A Finite Difference DSL for seismic imaging

- Aimed at creating fast high-order inversion kernels
- Development is driven by real-world problems

Based on SymPy expressions

- The acoustic wave equation:

$$m\frac{\partial^2 u}{\partial t^2} + \eta\frac{\partial u}{\partial t} - \nabla u = 0 \tag{1}$$

can be written as

```
eqn = m * u.dt2 + eta * u.dt - u.laplace
```

Devito auto-generates optimized C code and provides the necessary plumbing to use it directly from Python

Devito

Real-world applications need more than PDE solvers

- File I/O and support for large datasets
- Non-PDE kernel code e.g. insert source, sample receivers
- Ability to easily interface with complex outer code

Devito follows the principle of graceful degradation

- Circumvent restrictions to the high-level API by customization
- Allows custom functionality in auto-generated kernels

Introduction

Motivation

Example

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Typical loop nest - Devito code

OPESCI

```
def forward(model, nt, dt, h, order=2):
    shape = model.shape
    m = DenseData(name="m", shape=shape, space_order=order)
    m.data[:] = model
    u = TimeData(name='u', shape=shape, time_order=2,
                 space_order=order)
    eta = DenseData(name='eta', shape=shape,
                    space_order=order)

    # Derive stencil from symbolic equation
    eqn = m * u.dt2 - u.laplace + eta * u.dt
    stencil = [Eq(u.forward, solve(eqn, u.forward)[0])]

    # Add source and receiver interpolation
    source = u.inject(src * dt^2 / m)
    receiver = rec.interpolate(u)

    # Create and execute operator kernel
    op = Operator(stencils=source + stencil  + receiver,
                  subs={s: dt, h: h})
    op.apply(t=nt)
```


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Generated Code

```
#pragma omp parallel
  {
    _MM_SET_DENORMALS_ZERO_MODE(_MM_DENORMALS_ZERO_ON);
    _MM_SET_FLUSH_ZERO_MODE(_MM_FLUSH_ZERO_ON);
  }
  #pragma omp parallel
  {
    for (int i4 = 0; i4<329; i4+=1)
    {
      struct timeval start_main, end_main;
      #pragma omp master
      gettimeofday(&start_main, NULL);
      {
        #pragma omp for schedule(static)
        for (int i1 = 8; i1<122; i1++)
          for (int i2 = 8; i2<122; i2++)
          {
            #pragma omp simd aligned(damp, m, u:64)
            for (int i3 = 8; i3<122; i3++)
            {
              u[i4][i1][i2][i3] = ((3.04F*damp[i1][i2][i3] - 2*m[i1][i2][i3])*u[i4 - 2][i1][i2][i3] - 1.12198912198912e-7F*(u[i4 - 1][i1][i2][i3 - 8] + u[i4 - 1][i1][i2][
                          i3 + 8] + u[i4 - 1][i1][i2 - 8][i3] + u[i4 - 1][i1][i2 + 8][i3] + u[i4 - 1][i1 - 8][i2][i3] + u[i4 - 1][i1 + 8][i2][i3]) + 2.34472828758543e-6F*(
                          u[i4 - 1][i1][i2][i3 - 7] + u[i4 - 1][i1][i2][i3 + 7] + u[i4 - 1][i1][i2 - 7][i3] + u[i4 - 1][i1][i2 + 7][i3] + u[i4 - 1][i1 - 7][i2][i3] + u[i4
                          - 1][i1 + 7][i2][i3]) - 2.39357679357679e-5F*(u[i4 - 1][i1][i2][i3 - 6] + u[i4 - 1][i1][i2][i3 + 6] + u[i4 - 1][i1][i2 - 6][i3] + u[i4 - 1][i1][
                          i2 + 6][i3] + u[i4 - 1][i1 - 6][i2][i3] + u[i4 - 1][i1 + 6][i2][i3]) + 1.60848360528361e-4F*(u[i4 - 1][i1][i2][i3 - 5] + u[i4 - 1][i1][i2][i3 +
                          5] + u[i4 - 1][i1][i2 - 5][i3] + u[i4 - 1][i1][i2 + 5][i3] + u[i4 - 1][i1 - 5][i2][i3] + u[i4 - 1][i1 + 5][i2][i3]) - 8.16808080808081e-4F*(u[i4
                          - 1][i1][i2][i3 - 4] + u[i4 - 1][i1][i2][i3 + 4] + u[i4 - 1][i1][i2 - 4][i3] + u[i4 - 1][i1][i2 + 4][i3] + u[i4 - 1][i1 - 4][i2][i3] + u[i4 - 1][
                          i1 + 4][i2][i3]) + 3.48504781144781e-3F*(u[i4 - 1][i1][i2][i3 - 3] + u[i4 - 1][i1][i2][i3 + 3] + u[i4 - 1][i1][i2 - 3][i3] + u[i4 - 1][i1][i2 +
                          3][i3] + u[i4 - 1][i1 - 3][i2][i3] + u[i4 - 1][i1 + 3][i2][i3]) - 1.43758222222222e-2F*(u[i4 - 1][i1][i2][i3 - 2] + u[i4 - 1][i1][i2][i3 + 2] + u
                          [i4 - 1][i1][i2 - 2][i3] + u[i4 - 1][i1][i2 + 2][i3] + u[i4 - 1][i1 - 2][i2][i3] + u[i4 - 1][i1 + 2][i2][i3]) + 8.21475555555556e-2F*(u[i4 - 1][
                          i1][i2][i3 - 1] + u[i4 - 1][i1][i2][i3 + 1] + u[i4 - 1][i1][i2 - 1][i3] + u[i4 - 1][i1][i2 + 1][i3] + u[i4 - 1][i1 - 1][i2][i3] + u[i4 - 1][i1 +
                          1][i2][i3]) + 4*m[i1][i2][i3]*u[i4 - 1][i1][i2][i3] - 4.23474709115646e-1F*u[i4 - 1][i1][i2][i3])/(3.04F*damp[i1][i2][i3] + 2*m[i1][i2][i3]);
            }
          }
      }
```

Generated Code

```
#pragma omp master
  {
    gettimeofday(&end_main, NULL);
    timings->main += (double)(end_main.tv_sec-start_main.tv_sec)+(double)(end_main.tv_usec-start_main.tv_usec)/1000000;
  }
  #pragma omp single
  {
    struct timeval start_post_stencils0, end_post_stencils0;
    gettimeofday(&start_post_stencils0, NULL);
    for (int p_src = 0; p_src < 1; p_src += 1)
    {
      u[i4][(int)(floor(5.0e-2F*src_coords[p_src][0])) + 40][(int)(floor(5.0e-2F*src_coords[p_src][1])) + 40][(int)(floor(5.0e-2F*src_coords[p_src][2])) + 40] =
                9.2416F*(-1.25e-4F*(float)(-2.0e+1F*(int)(floor(5.0e-2F*src_coords[p_src][0])) + src_coords[p_src][0])*(float)(-2.0e+1F*(int)(floor(5.0e-2F*
                src_coords[p_src][1])) + src_coords[p_src][1])*(float)(-2.0e+1F*(int)(floor(5.0e-2F*src_coords[p_src][2])) + src_coords[p_src][2]) + 2.5e-3F*(float)
                (-2.0e+1F*(int)(floor(5.0e-2F*src_coords[p_src][0])) + src_coords[p_src][0])*(float)(-2.0e+1F*(int)(floor(5.0e-2F*src_coords[p_src][1])) + src_coords
                [p_src][1]) + 2.5e-3F*(float)(-2.0e+1F*(int)(floor(5.0e-2F*src_coords[p_src][0])) + src_coords[p_src][0])*(float)(-2.0e+1F*(int)(floor(5.0e-2F*
                src_coords[p_src][2])) + src_coords[p_src][2]) - 5.0e-2F*(float)(-2.0e+1F*(int)(floor(5.0e-2F*src_coords[p_src][0])) + src_coords[p_src][0]) + 2.5e-3
                F*(float)(-2.0e+1F*(int)(floor(5.0e-2F*src_coords[p_src][1])) + src_coords[p_src][1])*(float)(-2.0e+1F*(int)(floor(5.0e-2F*src_coords[p_src][2])) +
                src_coords[p_src][2]) - 5.0e-2F*(float)(-2.0e+1F*(int)(floor(5.0e-2F*src_coords[p_src][1])) + src_coords[p_src][1]) - 5.0e-2F*(float)(-2.0e+1F*(int)(
                floor(5.0e-2F*src_coords[p_src][2])) + src_coords[p_src][2]) + 1)*src[i4][p_src]/m[(int)(floor(5.0e-2F*src_coords[p_src][0])) + 40][(int)(floor(5.0e
                -2F*src_coords[p_src][1])) + 40][(int)(floor(5.0e-2F*src_coords[p_src][2])) + 40] + u[i4][(int)(floor(5.0e-2F*src_coords[p_src][0])) + 40][(int)(
                floor(5.0e-2F*src_coords[p_src][1])) + 40][(int)(floor(5.0e-2F*src_coords[p_src][2])) + 40];
       u[i4][(int)(floor(5.0e-2F*src_coords[p_src][0])) + 40][(int)(floor(5.0e-2F*src_coords[p_src][1])) + 41][(int)(floor(5.0e-2F*src_coords[p_src][2])) + 40] =
                 9.2416F*(1.25e-4F*(float)(-2.0e+1F*(int)(floor(5.0e-2F*src_coords[p_src][0])) + src_coords[p_src][0])*(float)(-2.0e+1F*(int)(floor(5.0e-2F*
                 src_coords[p_src][1])) + src_coords[p_src][1])*(float)(-2.0e+1F*(int)(floor(5.0e-2F*src_coords[p_src][2])) + src_coords[p_src][2]) - 2.5e-3F*(float)
                 (-2.0e+1F*(int)(floor(5.0e-2F*src_coords[p_src][0])) + src_coords[p_src][0])*(float)(-2.0e+1F*(int)(floor(5.0e-2F*src_coords[p_src][1])) +
                 src_coords[p_src][1]) - 2.5e-3F*(float)(-2.0e+1F*(int)(floor(5.0e-2F*src_coords[p_src][1])) + src_coords[p_src][1])*(float)(-2.0e+1F*(int)(floor(5.0
                 e-2F*src_coords[p_src][2])) + src_coords[p_src][2]) + 5.0e-2F*(float)(-2.0e+1F*(int)(floor(5.0e-2F*src_coords[p_src][1])) + src_coords[p_src][1]))*
                 src[i4][p_src]/m[(int)(floor(5.0e-2F*src_coords[p_src][0])) + 40][(int)(floor(5.0e-2F*src_coords[p_src][1])) + 41][(int)(floor(5.0e-2F*src_coords[
                 p_src][2])) + 40] + u[i4][(int)(floor(5.0e-2F*src_coords[p_src][0])) + 40][(int)(floor(5.0e-2F*src_coords[p_src][1])) + 41][(int)(floor(5.0e-2F*
                 src_coords[p_src][2])) + 40];
      u[i4][(int)(floor(5.0e-2F*src_coords[p_src][0])) + 41][(int)(floor(5.0e-2F*src_coords[p_src][1])) + 40][(int)(floor(5.0e-2F*src_coords[p_src][2])) + 40] =
                9.2416F*(1.25e-4F*(float)(-2.0e+1F*(int)(floor(5.0e-2F*src_coords[p_src][0])) + src_coords[p_src][0])*(float)(-2.0e+1F*(int)(floor(5.0e-2F*src_coords
                [p_src][1])) + src_coords[p_src][1])*(float)(-2.0e+1F*(int)(floor(5.0e-2F*src_coords[p_src][2])) + src_coords[p_src][2]) - 2.5e-3F*(float)(-2.0e+1F*(
                int)(floor(5.0e-2F*src_coords[p_src][0])) + src_coords[p_src][0])*(float)(-2.0e+1F*(int)(floor(5.0e-2F*src_coords[p_src][1])) + src_coords[p_src][1])
                 - 2.5e-3F*(float)(-2.0e+1F*(int)(floor(5.0e-2F*src_coords[p_src][0])) + src_coords[p_src][0])*(float)(-2.0e+1F*(int)(floor(5.0e-2F*src_coords[p_src
                ][2])) + src_coords[p_src][2]) + 5.0e-2F*(float)(-2.0e+1F*(int)(floor(5.0e-2F*src_coords[p_src][0])) + src_coords[p_src][0]))*src[i4][p_src]/m[(int)(
                floor(5.0e-2F*src_coords[p_src][0])) + 41][(int)(floor(5.0e-2F*src_coords[p_src][1])) + 40][(int)(floor(5.0e-2F*src_coords[p_src][2])) + 40] + u[i4
                ][(int)(floor(5.0e-2F*src_coords[p_src][0])) + 41][(int)(floor(5.0e-2F*src_coords[p_src][1])) + 40][(int)(floor(5.0e-2F*src_coords[p_src][2])) + 40];
      u[i4][(int)(floor(5.0e-2F*src_coords[p_src][0])) + 40][(int)(floor(5.0e-2F*src_coords[p_src][1])) + 40][(int)(floor(5.0e-2F*src_coords[p_src][2])) + 41] =
                9.2416F*(1.25e-4F*(float)(-2.0e+1F*(int)(floor(5.0e-2F*src_coords[p_src][0])) + src_coords[p_src][0])*(float)(-2.0e+1F*(int)(floor(5.0e-2F*src_coords
```

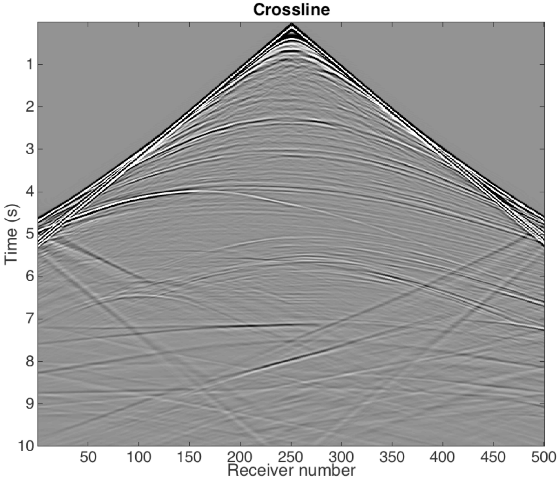
Test on SEAM dataset

Full SEAM:

- 10m grid
- 5001 time steps (10sec modelling)
- 1575x1575x1125 domain
- 10Hz source in the middle of XY 40m depth
- 500x500 receivers
- Intel(R) Xeon(R) E5-2680 v2, 10 cores per socket, dual socket
- Time for a single modeling: 4h15min

Test on SEAM dataset

Shot Record



Test on SEAM dataset

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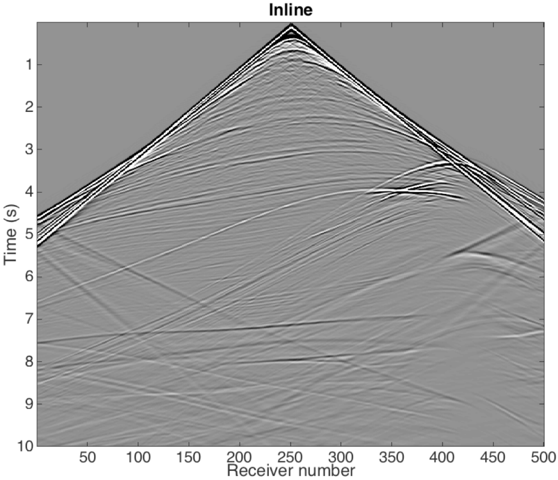
Shot Record



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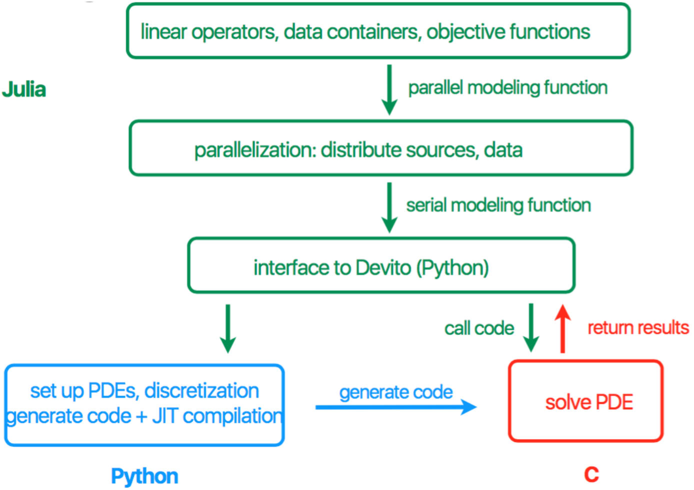
Vertical Integration

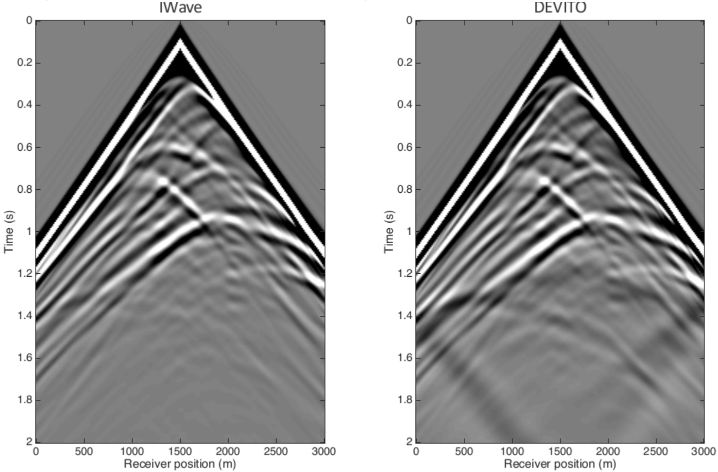


[1]

[1] Philipp Witte, Mathias Louboutin, and Felix J. Herrmann. Large-scale workflows for wave-equation based inversion in julia. In *Domain-Specific Abstractions for Full-Waveform Inversion at SIAM CSE*, 2017

Verification

Comparison with a reference implementation - IWAVE



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Verification of the generated code:

- Extensive unit-testing already in place with continuous integration (Travis)
- Adjoint test
  - For any $x \in$span$(P_s A^T P_r^T)$, $y \in$span$(P_r A^T P_s^T)$
  - $< P_r A^T P_s^T x, y > - < x P_s A^T P_r^T y >= 0$
  - Passes with at-least 8 matching significant digits for 2D and 3D with 2,4,6,8,10,12th order discretization
- Gradient test
  - For a small model perturbation dm, $\phi_s(m + hdm) = \phi_s(m) + \mathcal{O}(h)$ and $\phi_s(m + hdm) = \phi_s(m) + h(J[m]^T \delta d)dm + \mathcal{O}(h^2)$
  - Passes at the level of the machine's accuracy
- Automatic formal code verification being implemented [1]

[1] Christopher Lidbury, Andrei Lascu, Nathan Chong, and Alastair F Donaldson. Many-core compiler fuzzing. In *ACM SIGPLAN Notices*, volume 50, pages 65–76. ACM, 2015

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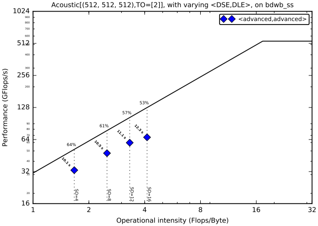
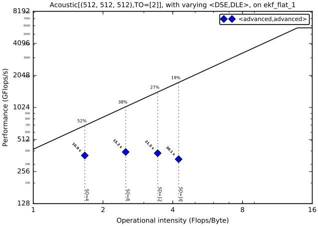
Performance

- Performance of acoustic forward operator
- Left:
  - Intel®Xeon™E5-2620 v4 2.1Ghz Broadwell (8 cores per socket, single socket)
  - Model size $512 \times 512 \times 512$, $t_n = 250$
- Right:
  - Intel®Xeon Phi™7650 Knightslanding (68 cores) Quadrant Mode
  - Model size $512 \times 512 \times 512$, $t_n = 3000$





Performance

- Performance of TTI[1] forward operator
- Left:
  - Intel®Xeon™E5-2620 v4 2.1Ghz Broadwell (8 cores per socket, single socket)
  - Model size $512 \times 512 \times 512$, $t_n = 250$
- Right:
  - Intel®Xeon Phi™7650 Knightslanding (68 cores) Quadrant mode
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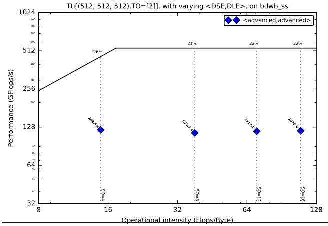

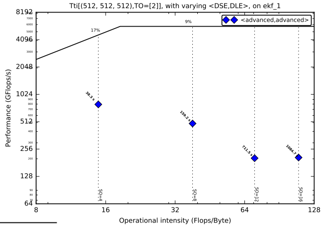

[1] Yu Zhang, Houzhu Zhang, and Guanquan Zhang. A stable tti reverse time migration and its implementation. *Geophysics*, 76(3):WA3–WA11, 2011

Conclusions

OPESCI

- Devito: A finite difference DSL for seismic imaging
  - Symbolic problem description (PDEs) via SymPy
  - Low-level API for kernel customization
  - Automated performance optimization
- Devito is driven by real-world scientific problems
  - Bring the latest in performance optimization closer to real science
- Future work:
  - Extend feature range to facilitate more science
  - MPI parallelism for larger models
  - Integrate stencil or polyhedral compiler backends like YASK

Thank you

Publications

- N. Kukreja, M. Louboutin, F. Vieira, F. Luporini, M. Lange, and G. Gorman. Devito: automated fast finite difference computation. WOLFHPC 2016
- M. Lange, N. Kukreja, M. Louboutin, F. Luporini, F. Vieira, V. Pandolfo, P. Velesko, P. Kazakas, and G. Gorman. Devito: Towards a generic Finite Difference DSL using Symbolic Python. PyHPC 2016
- M. Louboutin, M. Lange, N. Kukreja, F. Herrmann, and G. Gorman. Performance prediction of finite-difference solvers for different computer architectures. Submitted to Computers and Geosciences, 2016

Web links

- www.opesci.org
- github.com/opesci

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