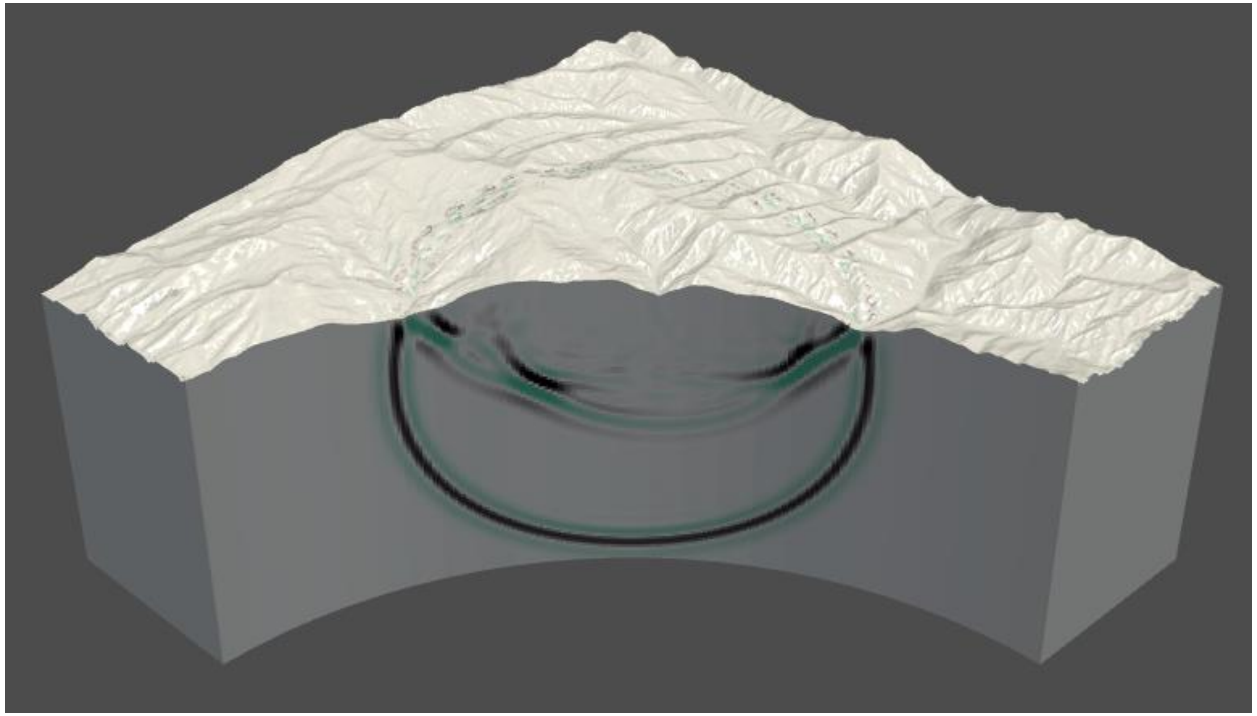
A flexible, high-level abstraction for topography implementation within finite difference wave solvers



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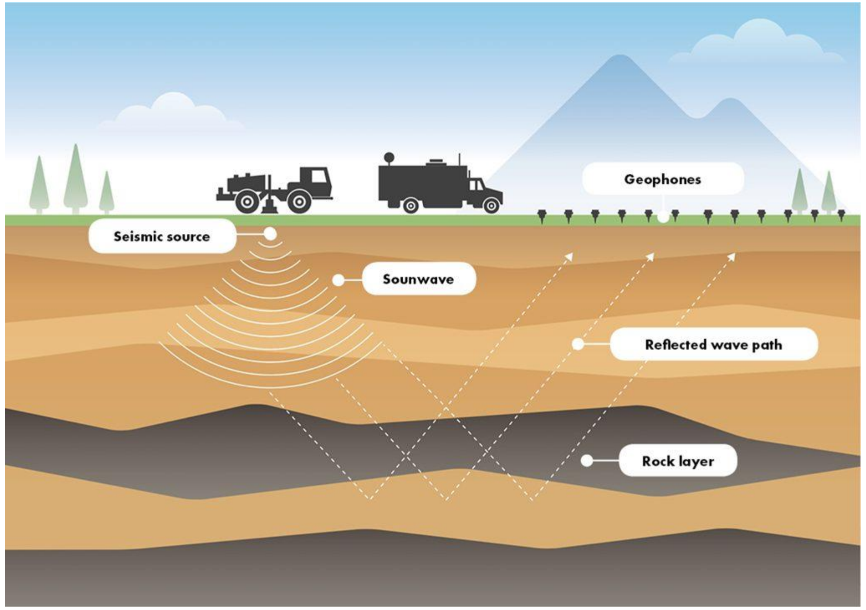
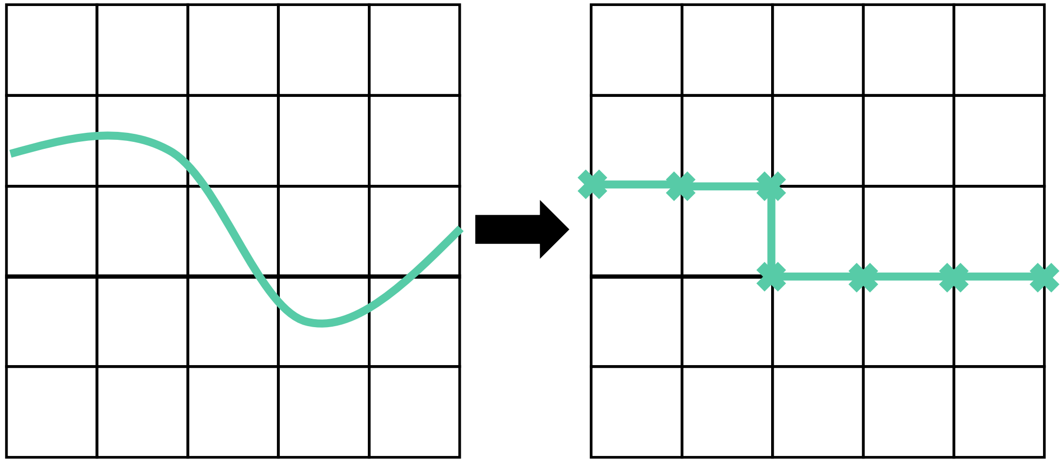
Motivation

- Seismic imaging/modelling with complex topography
  - Challenge of irregular topography on structured grids[1][2]
  - Sophisticated free-surface handling[4] required
- Curvilinear boundaries introduce additional complexity
- We present a high-level abstraction for immersed boundary specification
- Benchmarking against conventional free-surfaces, and wavefield models in mountainous terrain produce realistic results





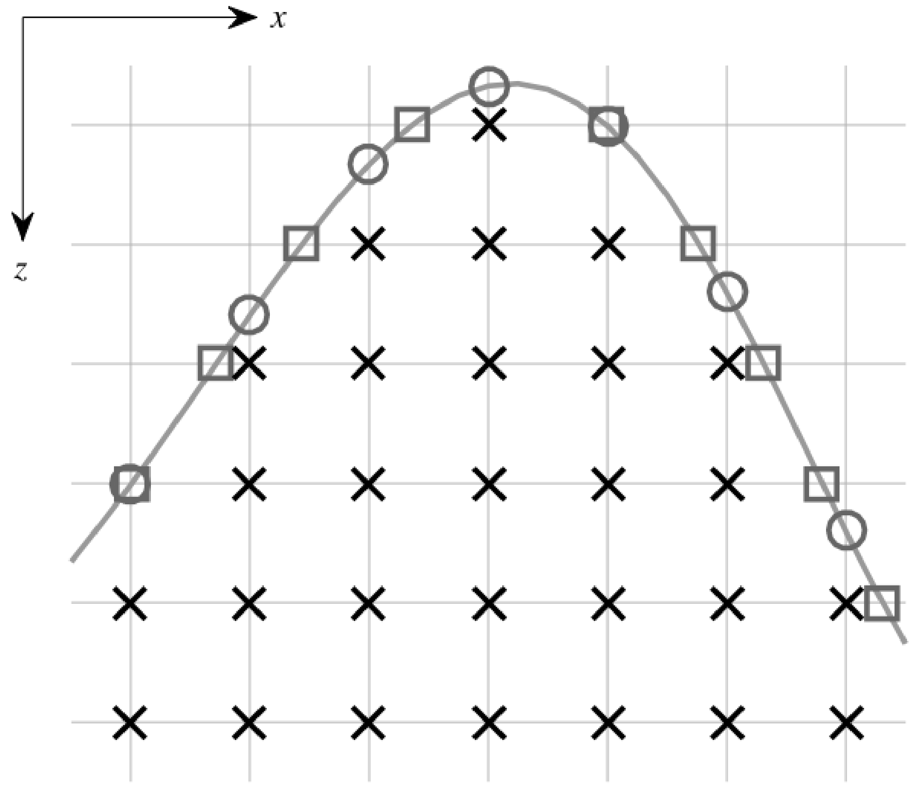
Staircasing of topography due to approximation to regular grid.

Abstracting the topography problem

- Numerous related problems:
  - Many wave equation formulations
  - Many space discretizations
  - Many boundary conditions
- Many implementations, but most share common components/concepts
- Leverage symbolic computation to generate schemes based on specification

Immersed boundaries

- Means of implementing surfaces of arbitrary shape within FD schemes
- Boundary conditions enforced off-grid by extrapolating solution
- Avoids curvilinear grids and other geometric transformations



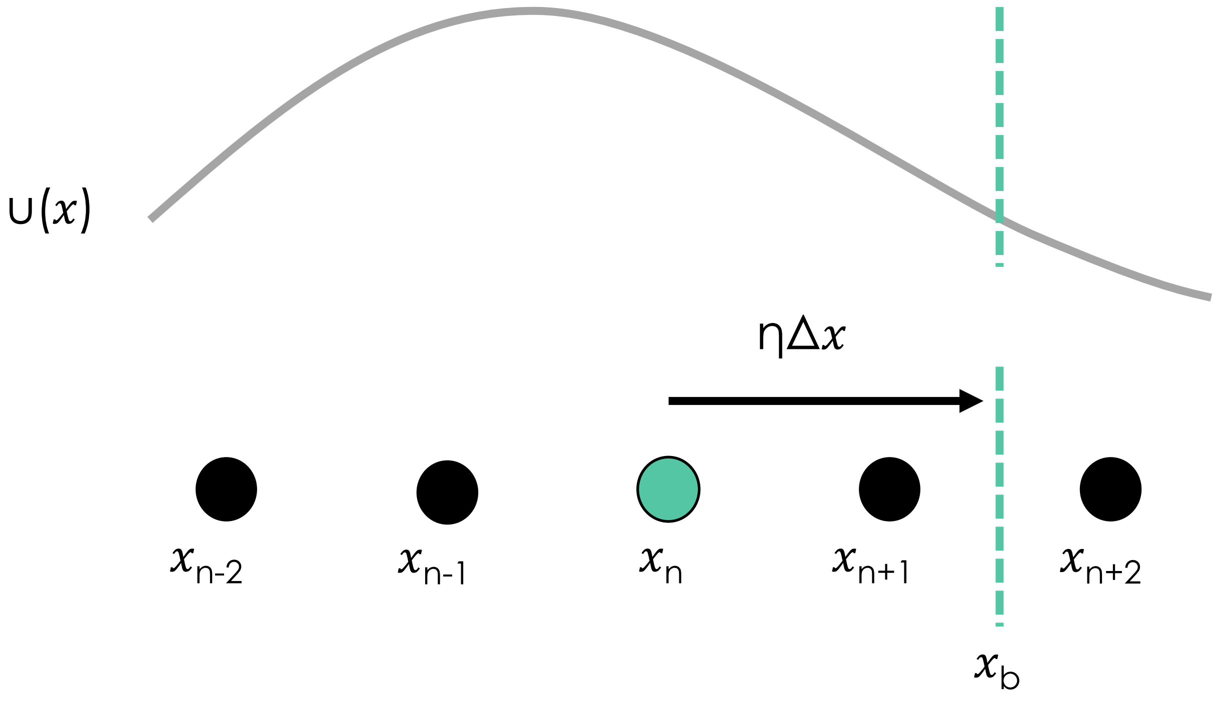
Immersed boundary representation relative to the finite difference grid[3]

Immersed boundary scheme

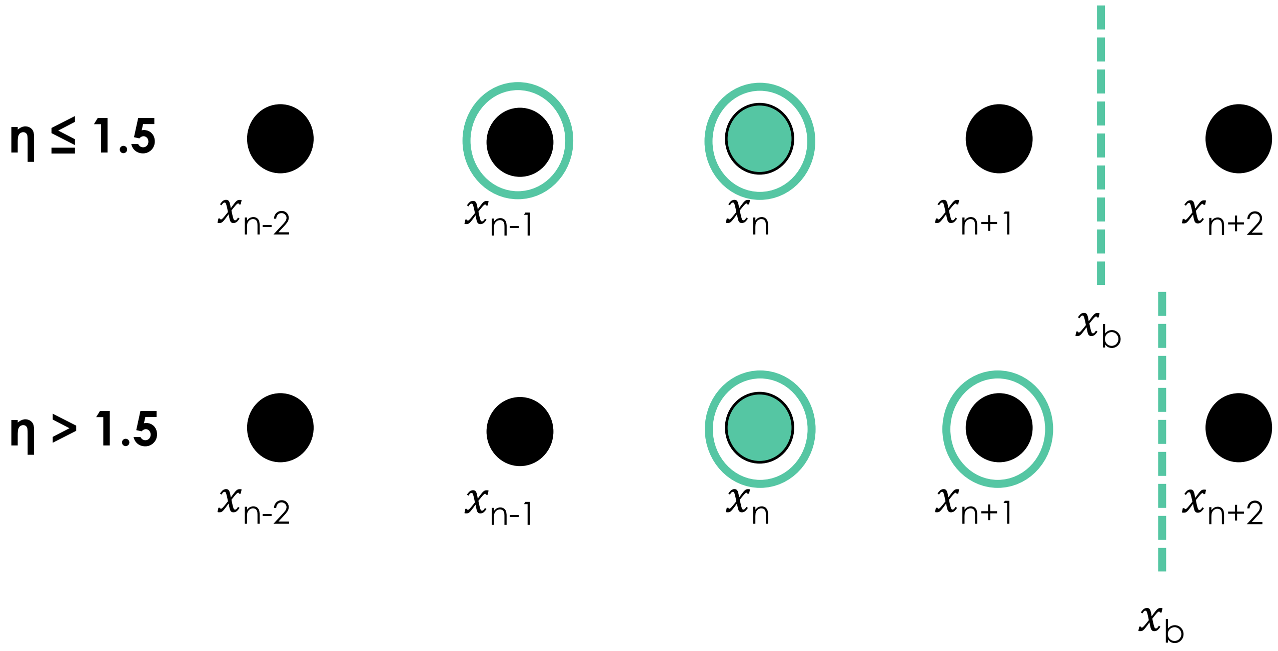
- Generalisation of Mulder 2017 scheme[3] with extrapolations automatically determined from BCs and discretisation
- One independent extrapolation per spatial dimension
  - Avoids potentially instability-inducing ambiguities[3]
- Boundary encapsulated by modified stencil coefficients
  - Ghost nodes are not required
  - Straightforward to locally modify for stability

Stencil modifications

- Consider a stencil of space order 4 intersecting a boundary at $x_b = x_n + \eta\Delta x$ where $0 < \eta \leq 2$
- $u(x)$ approximated as order 4 polynomial
- Substituting polynomial into boundary conditions produces a system of linear equations
- This system is solved to obtain the extrapolation polynomial



Stencil modifications



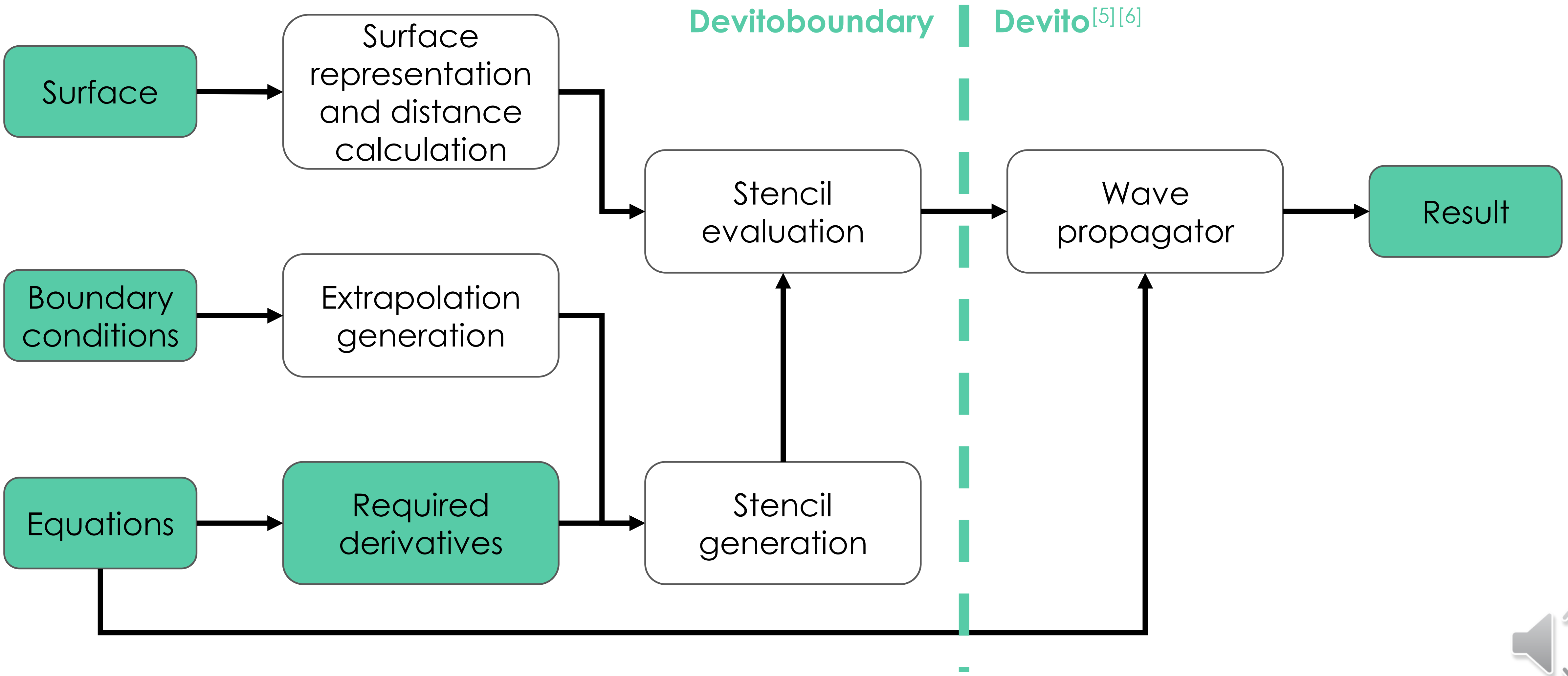
Stencil points used for extrapolation in the two possible cases: circles denote points used in the extrapolation

- Stencil points within half a grid spacing of the boundary excluded
- The polynomial is evaluated at exterior points
- Extrapolated values of u are substituted into the stencil, eliminating exterior points

Generalisation

- Straightforward to extend to other discretizations or boundary conditions
- Where stencils are severely truncated at both ends, a higher order extrapolation may be used to honour BCs
- For any combination of boundary conditions, discretization, and derivative, there are $(\text{order}+1)^2$ stencil variants

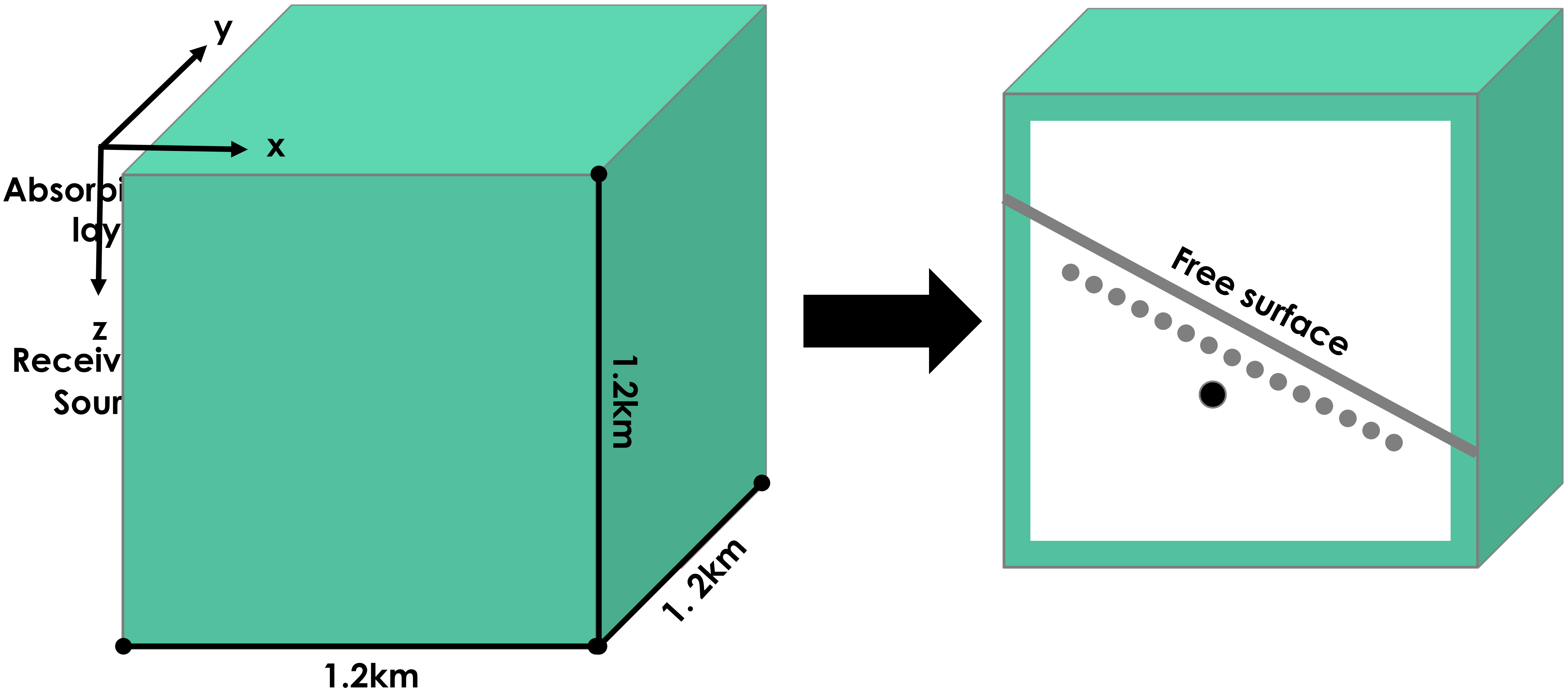
The Devitoboundary pipeline



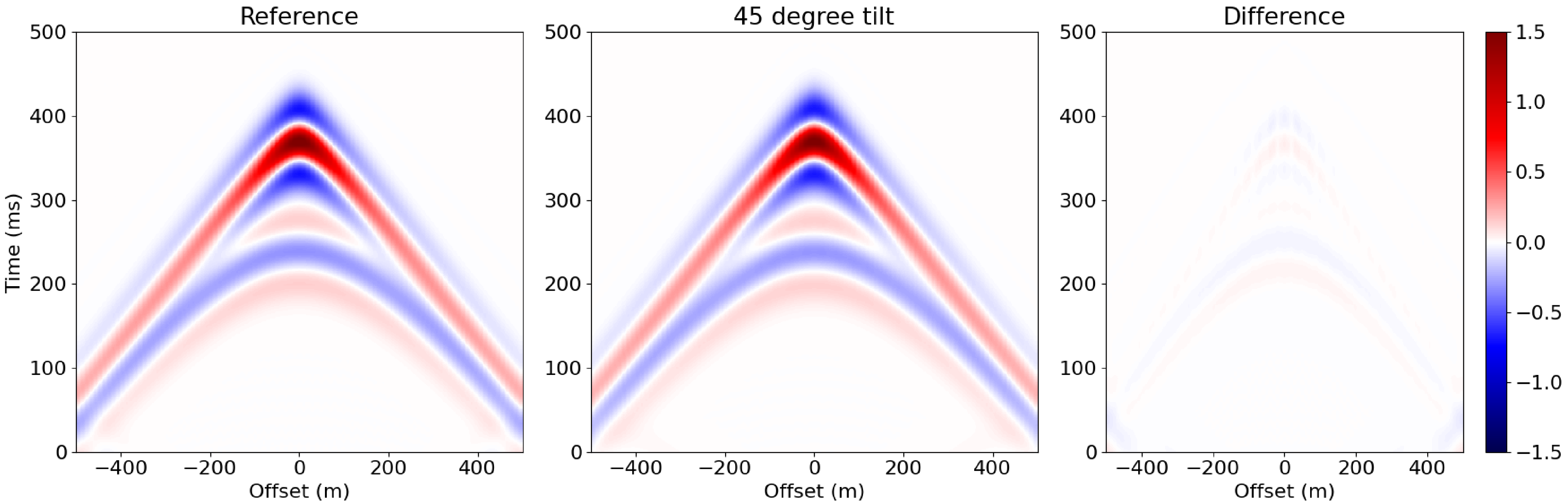
Computational aspects

- Stencil generation limited to pre-processing step
- Devitoboundary MPI/Dask support planned
- Devito kernel produces highly-optimized low-level code
  - SIMD, OpenMP, OpenACC, MPI, …
  - FLOP and memory optimizations
- Performance optimizations required before meaningful comparisons can be made

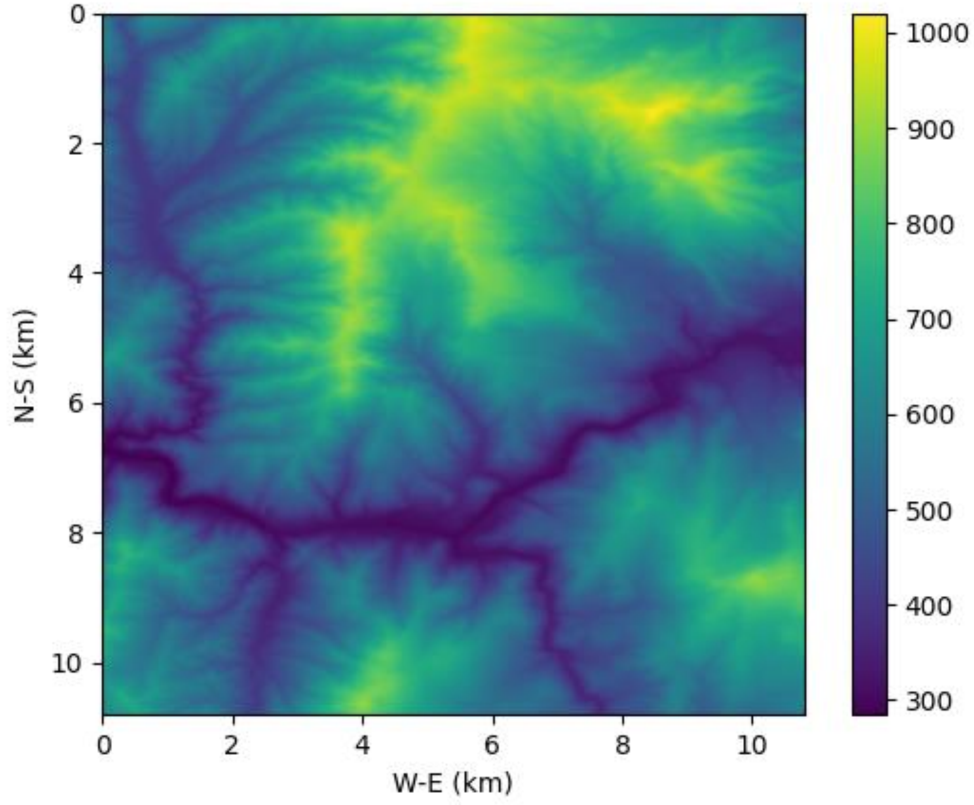
Validation of pipeline



Validation of pipeline



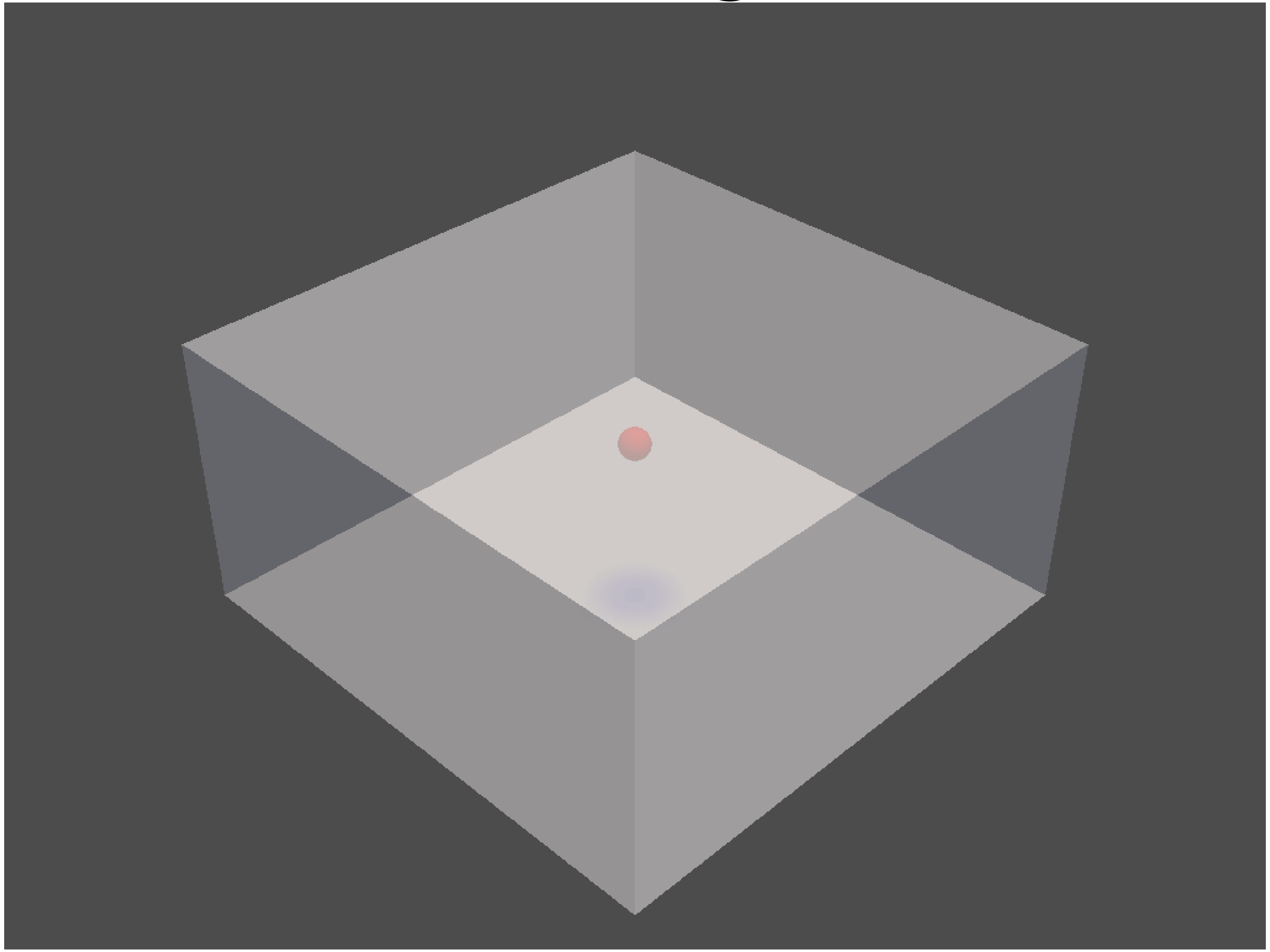
Demo: forward model with topography



The raster Digital Elevation Model (DEM) used to specify the boundary surface implemented in the model. Colourbar shows elevation in meters.

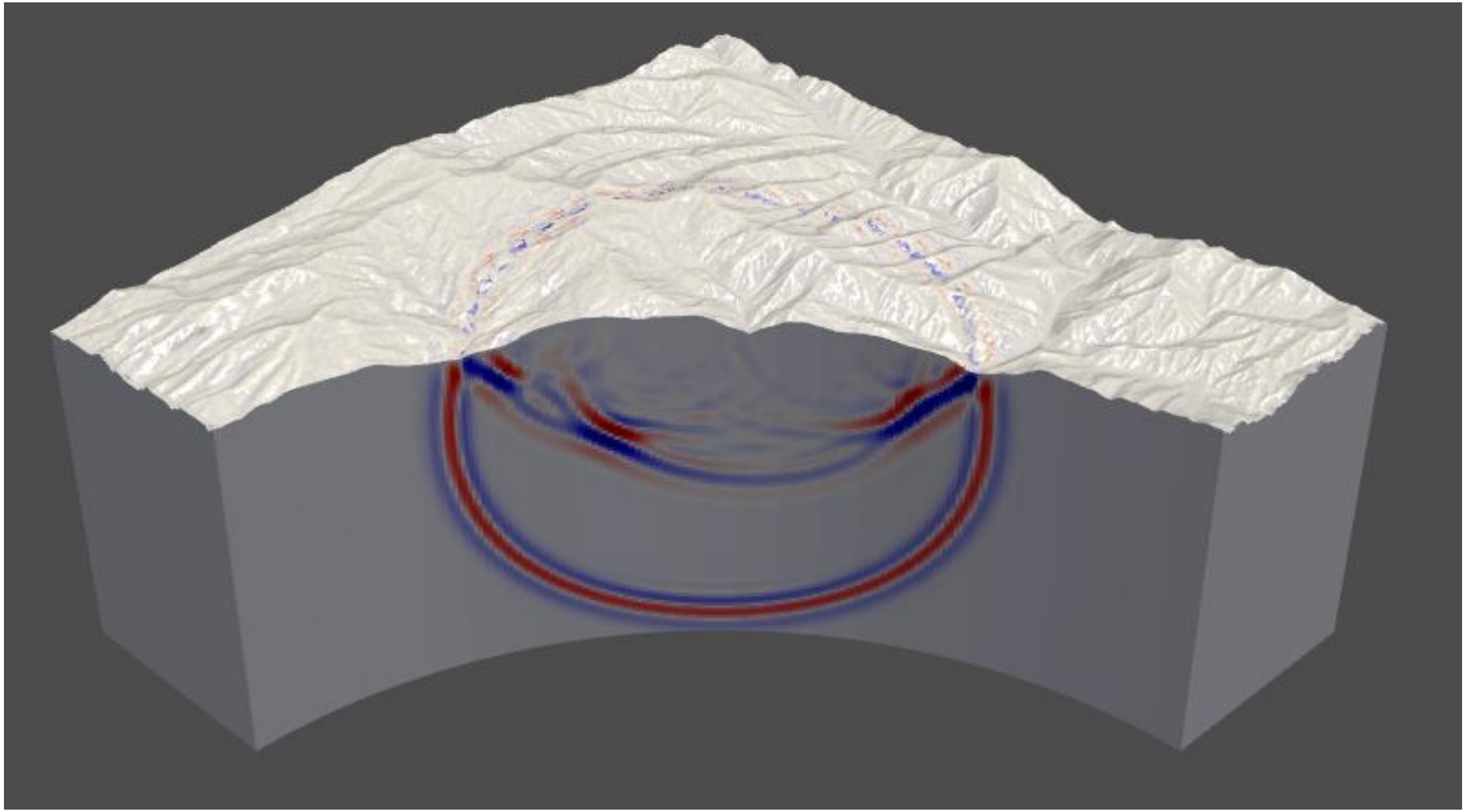
- Umpqua National Forest – Oregon
  - Top left corner at 43°N, 123 °W
  - 10.8km x 10.8km surface from 1 arcsecond SRTM DEM
- Mountainous terrain (~700m variation)
- Complex surface geometry to handle

Model Configuration

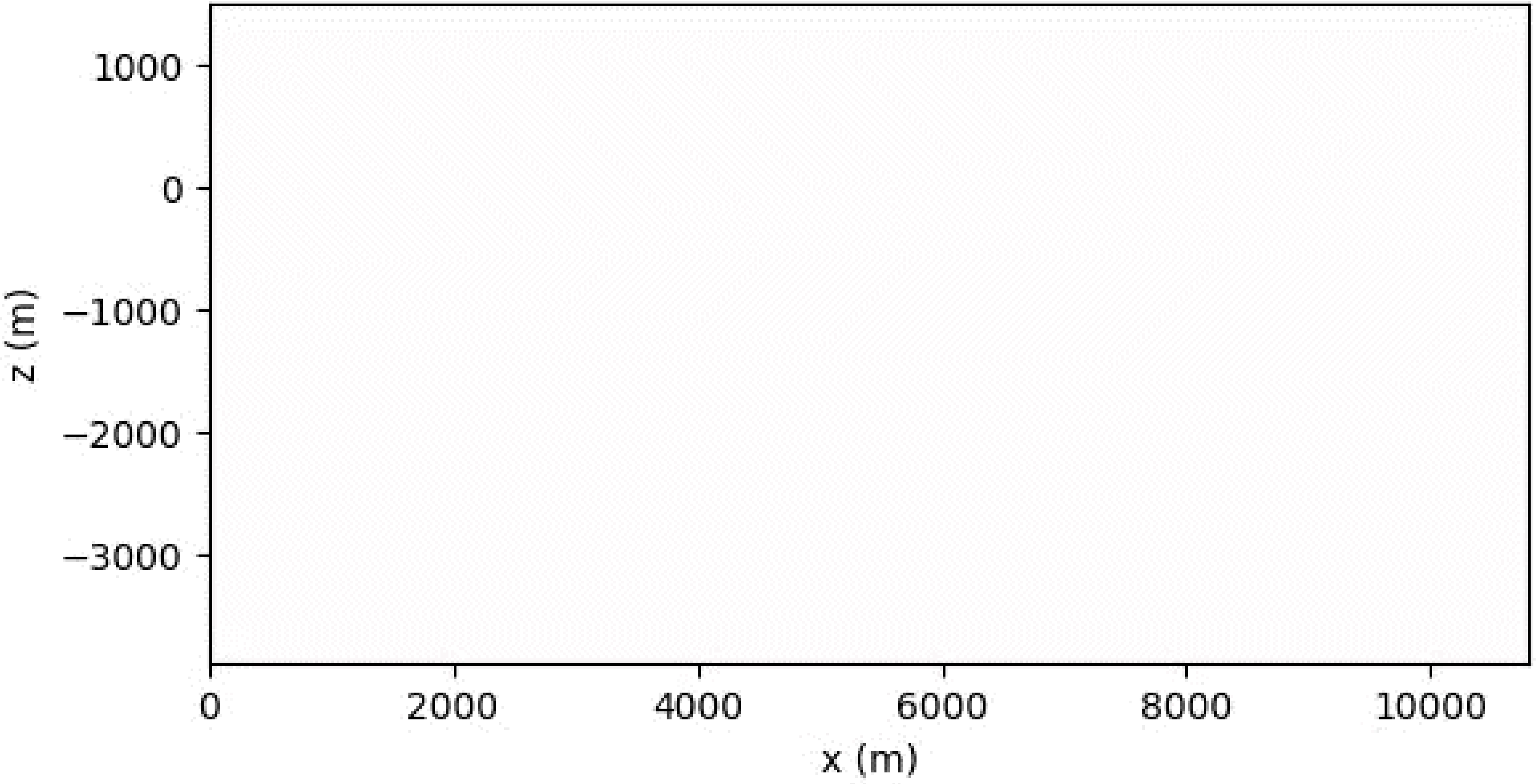


- 10.8km x 10.8km x 5.4km grid, 50m spacing
- Homogenous P-wave velocity of 1.2km/s
- 4^{th} order in space, 2^{nd} in time
- Free surface
- Ricker source positioned centrally, 500m below sea level

Results



Results



A slice through a 3D model of seismic waves interacting with mountainous topography, made using Devito and Devitoboundary

Summary

- High-level abstractions and symbolic computation enable a high degree of generality
- A simple immersed boundary scheme implemented with variable stencil coefficients enables flexible, stable, and accurate boundary representation
- Validation and test case show expected wavefield behaviour in the presence of non-grid-aligned topography

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Devitoboundary –
https://github.com/devitocodes/devitoboundary
Devito –
https://github.com/devitocodes/devito
https://www.devitoproject.org/
Devitocodes Slack –
devitocodes.slack.com/

References

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[2] Zhebel, E., Minisini, S., Kononov, A., & Mulder, W. A. (2014). A comparison of continuous mass-lumped finite elements with finite differences for 3-D wave propagation. Geophysical Prospecting, 63(1), 1111–1125.

[3] Mulder, W. A. (2017) 'A simple finite-difference scheme for handling topography with the second-order wave equation', Geophysics, 82(3), pp. 111–120.

[4] Gao, L. et al. (2015) 'An immersed free-surface boundary treatment for seismic wave simulation', Geophysics, 80(5), pp. 193–209.

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[6] Louboutin, M. et al. (2019) 'Devito (v3.1.0): an embedded domain-specific language for finite differences and geophysical exploration', Geoscientific Model Development, 12(3), pp. 1165–1187.